

King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math 605: Asymptotic Expansions and Perturbation Methods
Final Exam-Part I: In Class
Instructor: Prof. B. Chanane

Name: _____ ID: _____

Thursday 12 January 2017 at 7 PM

Problem 1 Use the Laplace method (not the formula) to obtain an asymptotic approximation of the integral

$$f(x) = \int_{-\infty}^{+\infty} \exp(-2\pi x \cosh t) dt, \quad x \rightarrow +\infty$$

Problem 2 Obtain an asymptotic solution of the equation

$$xy'' + 2(1-x)y' - y = 0$$

for $x \rightarrow +\infty$ in the form

$$y(x) \approx e^{\lambda x} x^\sigma \left(\alpha_0 + \frac{\alpha_1}{x} + \dots \right)$$

Problem 3 Use the WKB method to find asymptotic approximations of the eigenvalues μ^2 and corresponding eigenfunctions for large μ^2 for the following eigenvalue problem,

$$\begin{cases} -\frac{d^2 y}{dx^2} = \mu^2(1+x^2)y \\ y(0) = 1, y(1) = 0 \end{cases}$$

(provide all details)