King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math605-01 / Instructor: Prof. Bilal Chanane
Major Exam I Duration 2hrs

Name:..ID:...

Question	**Points**	**Mark**
1	**30**	
2	**30**	
3	**20**	
4	**20**	
Total	**100**	

Exercise 1 *Consider the integral*

$$f(x) = \int_0^\infty f(t) e^{x\phi(t)} dt$$

with $f(t) = \exp(-\frac{1}{t})$ *and* $\phi(t) = -t$.

(a) Show that Watson's lemma cannot be applied directly to determine the leading behaviour of $f(x)$ *as* $x \to \infty$. *Justify your answer.*

(b) Find the location t *of the maximum of the integrand. Note that it is a movable maximum since* t *depends on* x,

(c) Use the change of variable $t = \frac{s}{\sqrt{x}}$ *then apply Laplace method to the resulting integral to determine this leading behaviour of* $f(x)$ *as* $x \to \infty$.

Exercise 2 *Use the method of stationary phase to obtain the leading behaviour of* $J_n(n)$ *as* $n \to \infty$ *where* J_n *ios the Bessel function which has the integral representation,*

$$J_n(x) = \frac{1}{\pi} \int_0^{\pi} \cos(x \sin t - nt) dt$$

Exercise 3 *Show that the sequence* $\{\sin^n x\}_{n\geq 0}$ *is asymptotic as* $x \to 0$, *then find the asymptotic series of* $f(x) = \tan x$ *with respect to these gauge functions as* $x \to 0$.

Exercise 4 *Find asymptotic expansions for the roots of the equation,*

$$\epsilon x^3 + x^2 + 2x - 3 = 0$$

as $\epsilon \to 0$ *correct to* $o(\varepsilon^2)$