King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
MATH533 - Complex Variables
Exam I – Semester 161

Exercise 1

Find all possible values of

$$\tan^{-1}(1+i)$$

Exercise 2

True or false (if true, give a short explanation, if false, give a counterexample)

1. If two entire functions agree at infinitely many points, then they must be equal.
2. An entire function bounded in a half-plane is constant.
3. There exists f analytic in $|z| < 1$ such that $f(1/n) = n/(2+n)$ for $n \in \mathbb{N}$.
4. An entire function $f = u + iv$ such that $u - v \leq 2016$ is constant

Exercise 3

Suppose that $G \subset \mathbb{C}$ is open, f is analytic in G, and $\gamma : [0,1] \to G$ is a smooth curve in G. Show that

$$\int_\gamma f'(z)dz = f(\gamma(1)) - f(\gamma(0)).$$

Exercise 4

Evaluate the following integrals

1. $\int_{|z|=3} \frac{3z^4 + 2z - 6}{(z-2)^3} dz$
2. $\int_{|z|=2} \frac{dz}{z^4 - 1}$

Exercise 5

Suppose $f : \mathbb{C} \to \mathbb{C}$ is not constant *entire* function. Show:

1. There is at least one z in $\mathbb{C}$ with $|f(z)| > 1$.
2. There is at least one z in $\mathbb{C}$ with $|f(z)| < 1$.
3. There is at least one z in $\mathbb{C}$ with $|f(z)| = 1$.

Exercise 6

For each of the following real valued functions $u(x, y)$ find a real valued function $v(x, y)$ such that the function $f(z) = f(x + iy) = u(x, y) + iv(x, y)$ is analytic or show that there can be no such function.

(a) $u(x, y) = x^3 - 3xy^2 - 2xy$

(b) $u(x, y) = x^3 - xy^2 - 2xy$

Exercise 7

Suppose that f is an entire function such that

$$|f(z)| \leq 3|z|^{3/2} \text{ for all } z \in \mathbb{C}, |z| \geq 1.$$

Prove that $f(z) = az + b$ for some $a, b \in \mathbb{C}$ such that $|a| + |b| \leq 3$.