King Fahd University of Petroleum & Minerals

Department of Mathematics and Statistics

Semester 161

Math 513 HW Assignment # 3

Due Date: November 22, 2016

1.

a) Prove that $\int_0^\infty e^{-z^2-\frac{k}{z^2}}dz = \frac{\sqrt{\pi}}{2}e^{-2\sqrt{k}}$.

b) The error function is defined as $erf(t) = \frac{2}{\sqrt{\pi}}\int_0^t e^{-z^2}dz$ while the complementary error function is defined as $erfc(t) = 1 - erf(t) = \frac{2}{\sqrt{\pi}}\int_t^\infty e^{-z^2}dz$. Prove that Laplace transform of $erfc(\sqrt{a/t})$, $a \geq 0$ is $\frac{1}{S}e^{-2\sqrt{aS}}$.

2. Evaluate

a) $\mathcal{L}\left[t\,e^{-3t}\cos 3t\right]$.

b) $\mathcal{L}^{-1}\left[\frac{s^2+1}{s(s-1)(s+1)(s-2)}\right]$

c) $\mathcal{L}^{-1}\left[\frac{e^{-2s}}{s^2(s-1)}\right]$

3. Use Laplace transform to solve $\frac{d^4y}{dt^4} = f(t)$ subject to the boundary conditions $y(0) = y'(0) = y(2) = y'(2) = 0$,

a) when $f(t) = \delta(t-1)$

b) when $f(t) = \begin{cases} 1-t & 0 < t \leq 1 \\ 0 & 1 < t \end{cases}$

4. Solve the integral equation $f(t) = e^t + e^t\int_0^t e^{-\tau}f(\tau)\,d\tau$.

5. Use Laplace transform to solve the system

$$\frac{dx}{dt} = 4x - 2y + 2H(t-1)$$
$$\frac{dy}{dt} = 3x - y + H(t-1)$$
$$x(0) = 0,\ y(0) = \frac{1}{2}$$