King Fahd University of Petroleum & Minerals

Department of Mathematics and Statistics

Semester 161

Math 513 HW Assignment # 1

Due Date: October 18, 2016

1. Consider the function $f(t)=e^t$ on $(-\pi,\pi)$. Find the Fourier series of $f(t)$. What will the series converge to at $t=2\pi$ *and* $t=3\pi$?

2. Consider the function $g(t)=t^4$ on $(1,2)$. Find the Fourier series of $g(t)$ [You may use a machine to evaluate the integrals]. Based on the Fourier series of $g(t)$ and the fact that $\sum_{n=1}^{\infty}\frac{1}{n^2}=\frac{\pi^2}{6}$, evaluate the sum $\sum_{n=1}^{\infty}\frac{1}{n^4}$.

3. Let h be a given number in the interval $(0,\pi)$ and $f(t)=\begin{cases}\frac{h-t}{h} & if \;\; 0\le t<h \\ 0 & if \;\; h<t<\pi\end{cases}$.
   a) Find the Fourier cosine series of $f(t)$.
   b) Find the Fourier sine series of $f(t)$.
   c) Discuss the convergence and sketch the series in (a) and (b).

4. Write the Fourier series $\sum_{n=1}^{\infty}\frac{(-1)^n}{2n-1}\cos\left(\frac{(2n-1)\pi\,\mathrm{t}}{2}\right)$ in both the cosine and sine phase angle form.

5. Find the complex Fourier series for the function in problem 1. Sketch the amplitude spectrum.

6. Using Fourier series to solve the differential equation $y''+4y=f(t)$, where $f(t)=\begin{cases}1 & if \;\; -\pi<t<0 \\ 2 & if \;\; 0<t<\pi\end{cases}$. [Tip: Use a vertical shift to quickly find the Fourier series of $f(t)$].

7. Consider $f(t)$ with the property that $f(t+\pi)=-f(t)$ for all t.
   a) Show that the function is periodic.
   b) Sketch an example of such a function.
   c) Show that all its even Fourier coefficients are zero (i.e. $a_0=a_2=a_4=a_6=\cdots=0, \quad b_2=b_4=b_6=\cdots=0$.