King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics
Math 513 Final Exam
The First Semester of 2016-2017 (161)

Time Allowed: 180 Minutes

Name: _____ ID#: _____

Section/Instructor: _____ Serial #: _____

- Mobiles and calculators are not allowed in this exam.
- Write all steps clear.

Question #	Marks	Maximum Marks
1		15
2		15
3		15
4		20
5		20
6		20
7		10
Total		115

Q:1 (15 points) Solve the Sturm- Liouville problem:

$$y'' + \lambda y = 0, \qquad y(0) + y'(0) = 0, \quad y(\pi) + y'(\pi) = 0.$$

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Q:2 (15 points) Solve the heat equation

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, \quad 0 < x < \pi, \quad t > 0$$

subject to the following initial and **non-homogeneous** boundary conditions

$$u(x,0) = 1,\ u(0,t) = T_0,\ u_x(\pi,t) = 0,\ 0 < x < \pi, t > 0,$$

where T_0 is any constant.

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Q:3 (15 points) Use Laplace transform method to solve the wave equation

$$\frac{\partial^2 u}{\partial x^2} + \sin\pi \mathrm{x}\ \sin \mathrm{wt} = \frac{\partial^2 \mathrm{u}}{\partial \mathrm{t}^2}, \quad 0 < \mathrm{x} < 1, \quad \mathrm{t} > 0$$

with the boundary and initial conditions

$$u(0,t) = 0,\ u(1,t) = 0, t > 0$$

$$u(x,0) = 0,\ \frac{\partial u}{\partial t}|_{t=0} = 0,\ 0 < x < 1.$$

Q:4 (20 points) Solve the Laplace equation by separation of variables

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, 0 < x < 1, 0 < y < 1,$$
$$u(x,0) = 0, \quad u(0,y) = 10\ y,$$
$$\frac{\partial u}{\partial x}|_{x=1} = -1, \quad u(x,1) = 0.$$

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Q:5 (20 points) Solve

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial z^2} = 0, \quad 0 < r < 1, z > 0$$

subject to the following boundary conditions

$$u_r|_{r=1} + hu(1, z) = 0, z > 0,$$
$$u(r, 0) = 4, \quad 0 < r < 1.$$

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Q:6 (20 points) Find the steady-state temperature in the sphere of radious C by solving

$$\frac{\partial^2 u}{\partial r^2} + \frac{2}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} + \frac{cot\theta}{r^2}\frac{\partial u}{\partial \theta} = 0, 0 < r < C, 0 < \theta < \pi$$

$$u(C, \theta) = cos(\theta), \quad 0 < \theta < \pi.$$

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Q:7 (10 points) Let $A = \begin{pmatrix} -1 & 2 & -2 \\ 2 & -1 & 2 \\ -2 & 2 & -1 \end{pmatrix}$

(a) Find the eigenvalues and eigenvectors of A

(b) Find an **orthogonal matrix P** that diagonalizes **A** and find the diagonal matrix $D = P^T AP$.