King Fahd University Of Petroleum & Minerals
Department Of Mathematics And Statistics
STAT460 : Time Series (161)
Final Exam **Tuesday January 17, 2017**

Name: **ID:**

Question Number	Full Mark	Marks Obtained
One	9	
Two	12	
Three	14	
Four	12	
Five	10	
Six	14	
Seven	6	
Eight	12	
Nine	22	
Ten	14	
Eleven	20	
Total	145	

Some Formulas:

(1) For an ARIMA(p, d, q) with constant term, $\mu = \frac{\theta_0}{1-\phi_1-\phi_2-\ldots-\phi_p}$.

(2) The Yule-Walker Equations:

$$\begin{gathered}\rho_1 = \phi_1 + \phi_2\rho_1 + \ldots + \phi_p\rho_{p-1} \\ \rho_2 = \phi_1\rho_1 + \phi_2 + \ldots + \phi_p\rho_{p-2} \\ \ldots \\ \rho_p = \phi_1\rho_{p-1} + \phi_2\rho_{p-2} + \ldots + \phi_p\end{gathered}$$

(3) For an AR(p), $\gamma_0 = \frac{\sigma_e^2}{1-\phi_1\rho_1-\phi_2\rho_2-\ldots-\phi_p\rho_p}$

(4) For an AR(1), $Var\left(e_t(l)\right) = \sigma_e^2\left[\frac{1-\phi^{2l}}{1-\phi^2}\right]$

Question.1 (**2+5+2=9-Points**)

Let $Y_t = e_t - \theta e_{t-1}^2$, where $\{e_t\}$ are *iid* $N(0, \sigma_e^2)$.

(a) Find the mean function of Y_t

(b) Find the variance function of Y_t

(c) Based on your findings in the previous two pars. Is $\{Y_t\}$ stationary? Why?

Question .2 **(6+6=12-Points)**

Assume that $\{Y_t\}$ is time series explained by a non constant linear time trend given by $\mu_t = \beta_0 + \beta_1 t$.

Using the least squares method it can be shown that $\hat{\beta}_1 = \dfrac{\sum_{t=1}^{n}(Y_t - \bar{Y})(t - \bar{t})}{\sum_{t=1}^{n}(t - \bar{t})^2}$

(a) Show that $\hat{\beta}_1$ can be written as $\hat{\beta}_1 = \dfrac{\sum_{t=1}^{n}(t - \bar{t})Y_t}{\sum_{t=1}^{n}(t - \bar{t})^2}$

(b) If $\{Y_t\}$ is an independent and identically distributed time series with $Var(Y_t) = \gamma_0$ for all $t = 1, 2, \ldots, n$, and $\sum_{t=1}^{n}(t - \bar{t})^2 = \dfrac{n(n^2 - 1)}{12}$. Show that $Var(\hat{\beta}_1) = \frac{12\gamma_0}{n(n^2-1)}$

Question.3 **(10+4=14-Points)**

Let $\{Y_t\}$ be an MA(2) time series given by: $Y_t = e_t + \theta_1 e_{t-1} + \theta_2 e_{t-2}$, where e_t is a white noise $(0, \sigma_e^2)$

(a) Derive the autocovariance function for any lag $k, k \geq 0$.

(b) Find the autocorrelation function for any lag $k, k \geq 0$.

Question.4 **(4+3+5=12-Points)**

Consider the following time series model. $Y_t = 12 + 1.70Y_{t-1} - 0.70Y_{t-2} + e_t - 0.25e_{t-1}$

(a) Identify the model as a specific ARIMA model, that is, find p, d, q and what are the value of the parameters $(\phi s, \text{and} \theta s)$

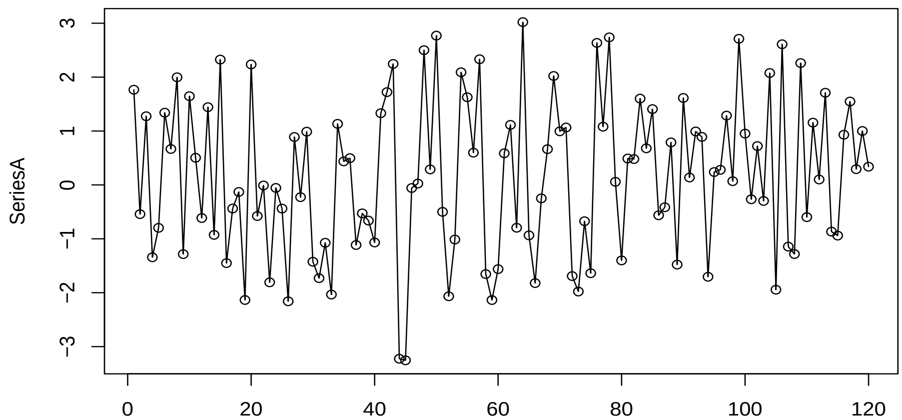
(b.) Find the $E\left(\nabla Y_t\right)$

(c.) Find $Var(\nabla Y_t)$

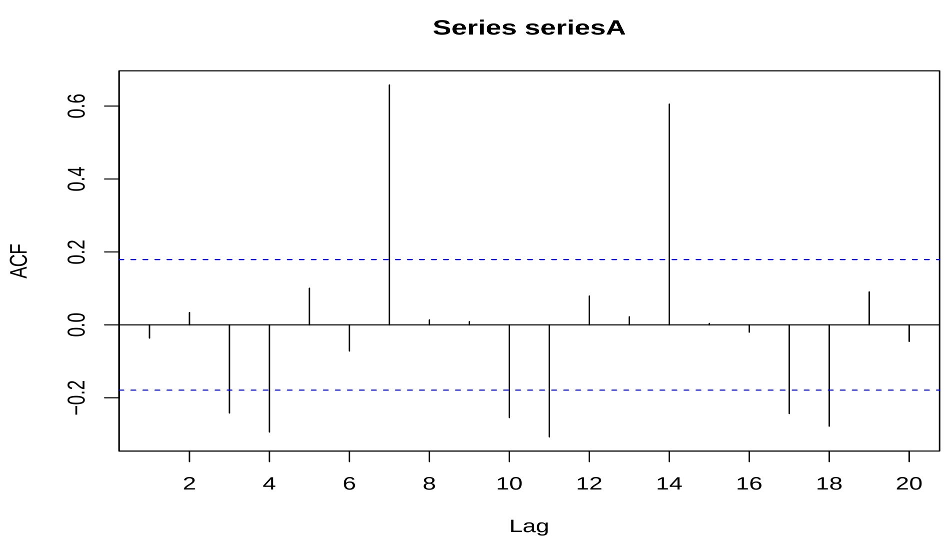
Question 5. (**2+2+2+4=10-Points**)

Consider the following plots of a time series named as SeriesA to answer the parts (a)-(d)

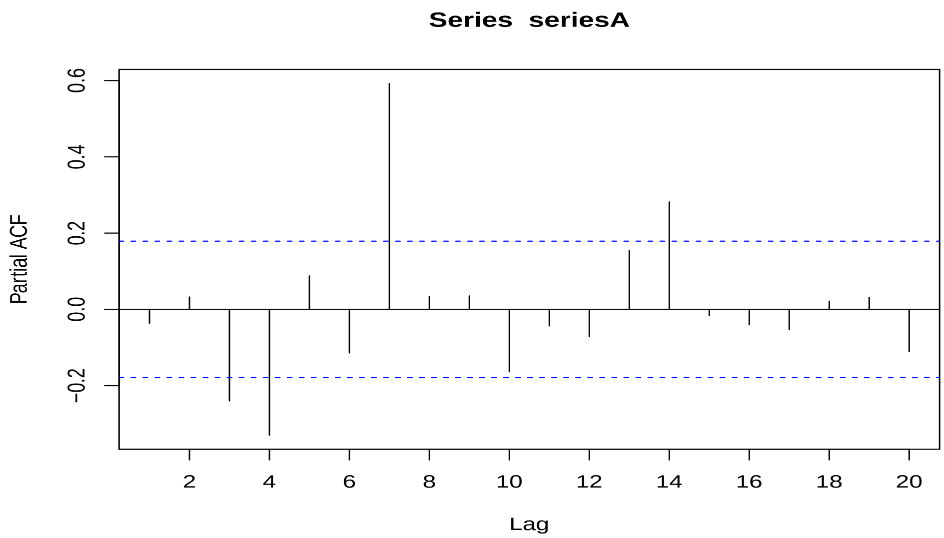
(a) A plot of the series against time. Does look a stationary time series? Why?



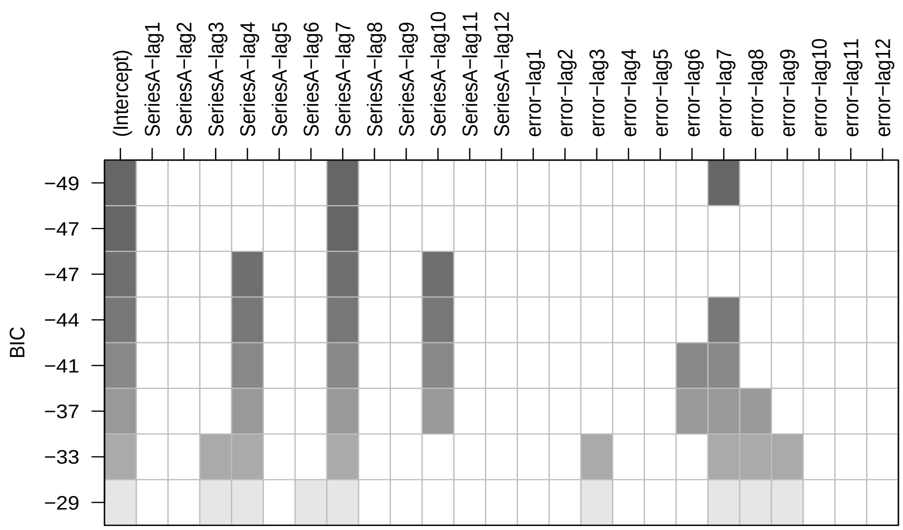
(b) The plot of the ACF, comment on the shape. Is there any specific model recommended for this series?



(c) The plot of the PACF, comment on the shape. Is there any specific model recommended for this series?



(c) The best subset ARMA selection based on BIC is given below. Write down the best model you can obtain for this series? Why it is the best one among others?



Question 6. (**6+8=14-Points**)

Consider the AR(2) process model with a constant term.

(a) Using the Yule-Walker equations, find the method of moments of ϕ_1 and ϕ_2.

(b) From a series of length $n = 120$, we have computed $r_1 = -0.851$, $r_2 = 0.774$, $\bar{Y} = 1.984$ and the sample variance 2.790. Find the method of moments estimates of ϕ_1 and ϕ_2, and an estimates of μ, θ_0 and σ_e^2.

Question 7. **(2+2+2 =6-Points)**

Use the following R-Output for simulated data of an AR(2) model to answer parts (a) to (c) below:

```
Autocorrelations of series seriesB, by lag

 1     2     3     4
 0.537 0.442 0.396 0.353
Call:
ar(x = seriesB, aic = F, order.max = 2, method = yw")"

Coefficients:
 1       2
0.4212  0.2159

Order selected 2  sigma^2 estimated as  1.401

Call:
  ar(x = seriesB, aic = F, order.max = 2, method = ols")"
 Coefficients:
   1       2
0.4680  0.1933

Intercept: 0.007193 (0.1457)

 Order selected 2  sigma^2 estimated as  1.23

Call: ar(x = seriesB, aic = F, order.max = 2, method = mle")"

Coefficients:
 1       2
0.4206  0.2084

Order selected 2  sigma^2 estimated as  1.325
```

(a) Find the maximum likelihood estimates of the parameters ϕ_1 and ϕ_2

(b) Find the method of moments estimates of the parameters ϕ_1 and ϕ_2

(c) Find the conditional least squares estimates of the parameters ϕ_1 and ϕ_2

Question 8. **(2+3+2+3+2 =12-Points)**

Use the following `R-Output` for simulated data of an AR(2) model to answer parts (a) to (c) below:

```
data(robot)
arima(robot,order=c(1,0,0))

Call:
arima(x = robot, order = c(1, 0, 0))
Coefficients:
      ar1  intercept
    0.3074     0.0015
s.e. 0.0528     0.0002

sigma^2 estimated as 6.482e-06:  log likelihood = 1475.54,  aic = -2947.08
******************************************************************************
arima(robot,order=c(0,0,1))

Call: arima(x = robot, order = c(0, 0, 1))

Coefficients:
        ma1  intercept
      0.2356     0.0015
s.e.  0.0477     0.0002

sigma^2 estimated as 6.658e-06:  log likelihood = 1471.22,  aic = -2938.45
******************************************************************************
arima(robot,order=c(0,1,1))
Call:
arima(x = robot, order = c(0, 1, 1))

Coefficients:
        ma1
       -0.8713
s.e.   0.0389

sigma^2 estimated as 6.069e-06:  log likelihood = 1480.95,  aic = -2959.9

******************************************************************************
arima(x = robot, order = c(1, 1, 0))

Call:
arima(x = robot, order = c(1, 1, 0))

Coefficients:
        ar1
       -0.4610
s.e.    0.0494

sigma^2 estimated as 7.815e-06:  log likelihood = 1440.71,  aic = -2879.42
```

(a) Estimate the parameters of an AR(1) model for this data, and write the estimated model assuming a zero mean.

(b) Test the significance of the model in part (a). Use $\alpha = 0.05$.

(c) Estimate the parameters of an IMA(1,1) model for this data, and write the estimated model assuming a zero mean.

(d) Test the significance of the model in part (c). Use $\alpha = 0.05$.

(e) Using the AIC criterion, which model you will choose from the previous two models?Why?

Question 9. **(4+4+4+4+3+3 =22-Points)**

Use the following `R-Output` for fitting an AR(3) model by maximum likelihood to the square root of the hare abundance series (file name :hare) to answer the parts (a)-(f).

```
data(hare)
model=arima(sqrt(hare),order=c(3,0,0))
Call:
arima(x = sqrt(hare), order = c(3, 0, 0))

Coefficients:
        ar1      ar2      ar3   intercept
       1.0519  -0.2292  -0.3931     5.6923
s.e.  0.1877   0.2942   0.1915     0.3371

sigma^2 estimated as 1.066:  log likelihood = -46.54,  aic = 101.08
*****************************************************************************
LB.test(model,lag=9)
Box-Ljung test

data:  residuals from  model
X-squared = 6.2475, df = 6, p-value = 0.396
*****************************************************************************
runs(rstandard(model))
$pvalue
[1] 0.602
$observed.runs
[1] 18
$expected.runs
[1] 16.09677
$n1
[1] 13
$n2
[1] 18
$k
[1] 0
*****************************************************************************

 Shapiro-Wilk normality test

data:  residuals(model)
W = 0.93509, p-value = 0.06043
```

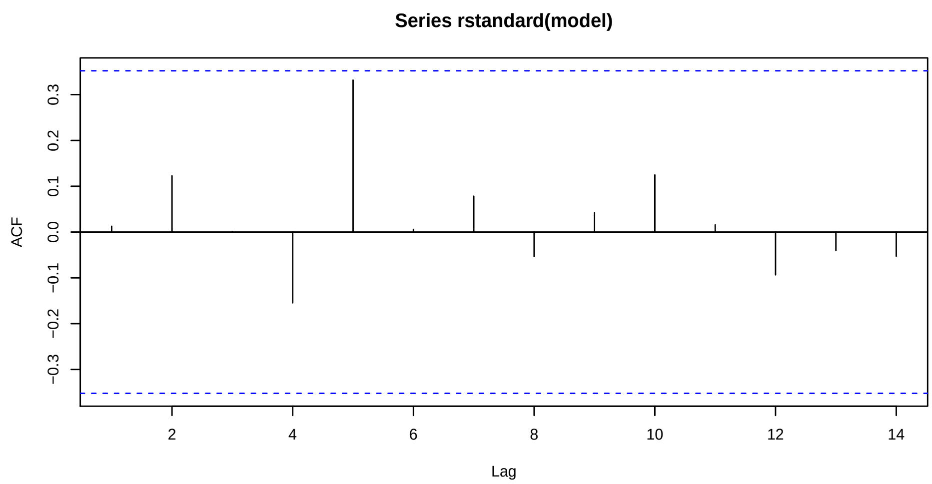
(a) Write down the estimated model. (Do not simplify)

(b) Obtain the Ljung-Box statistic summing to $K = 9$. Dose this statistic support the AR(3) specification? Explain. Use $\alpha = 0.05$.

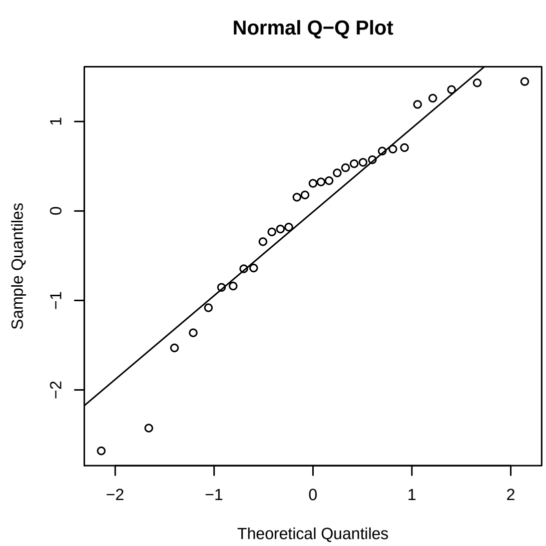
(c) Test the independence of the residuals using the `runs test`. Use $\alpha = 0.05$.

(d) Test the normality of standardised residuals of the fitted model. Use $\alpha = 0.05$.

(e) Below is the plot of the ACF of the residuals. Comment on the size of the correlations.



(f) The quantile-quantile normal plot of the residuals is given below, what result can you obtain? Explain.



Question 10. **(6+6+2 =14-Points)**

Consider an ARMA(1,1) model given by: $Y_t = \phi Y_{t-1} + \theta_0 + e_t - \theta e_{t-1}$

(a) Derive a formula for one step forecast value.

(b) For the model above, assume that $Y_t = 3.5$, $\phi = -0.45$, $\theta_0 = 6.9$ and $\theta = 0.25$. Find the forecasts for $t = 11$ and $t = 12$.

(c) If the actual value at $t = 11$, $t = 12$ are respectively 5.5 and 4.35. Find the forecast error for the two times.

Question 11. **(6+8+3+3 =20-Points)**

The following `R-output` is for fitting an AR(1) model for 52 simulated observations. The last observation in this series is `Yt=100.0858`.

```
Call:
arima(x = series, order = c(1, 0, 0))


Coefficients:
        ar1      intercept
      -0.8289    100.0031
s.e.   0.0715      0.0804

sigma^2 estimated as 1.105:  log likelihood = -76.95,  aic = 157.9
```

(a) Find the forecast values for lead times $l = 1$ and $l = 2$

(b) Find a 95% prediction interval for the forecast value when $l = 1$, and $l = 2$

(c) If the series value when $t = 1$ is 99.25. Update your forecast for lead time $l = 2$.

(c) The plot of the forecasts together with 95% forecast limits. Do the forecast values fall within the forecast limits?