**King Fahd University Of Petroleum & Minerals**
**Department Of Mathematics And Statistics**
**STAT460 : Time Series (161)**
**Second Exam** **Wednesday December 14, 2016**
**Name:** **ID:**

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| Question Number | Full Mark | Marks Obtained |
|---|---|---|
| One | 8 | |
| Two | 18 | |
| Three | 17 | |
| Four | 10 | |
| Five | 12 | |
| Six | 11 | |
| Seven | 12 | |
| Eight | 12 | |
| Total | 100 | |

Question.1 (**2+2+2+2=8-Points**) Define the following:

(a) A general linear process $\{Y_t\}$ :

(b) Partial autocorrelation function at lag $k$, $\phi_{kk}$

(c) ARIMA model

(d) Akaike's Information Criterion (AIC):

Question .2 **(4+7+7=18-Points)**

Suppose that $\{Y_t\}$ is an $AR(1)$ process given by: $Y_t = \phi Y_t + e_t$, with $-1 < \phi < 1$. Then

(a) Find the variance of this time series, $\gamma_0$.

(b) Show that $\gamma_k = \phi^k \gamma_0$

(c) If $W_t = \nabla Y_t$, prove that the covariance function for $W_t$ is $-[\frac{1-\phi}{1+\phi}]\phi^{k-1}\sigma_e^2$

Question.3 **(4+5+8=17-Points)**

Let $\{Y_t\}$ be a time series given by: $Y_t = 0.7Y_{t-1} + e_t + 0.2e_{t-1} + 0.3e_{t-2}$, with $E(Y_t) = 0$.

(a.) Identify the model and write all parameters, (i.e the $\phi$'s and $\theta$'s)

(b.) Show that $\rho_k = 0.7\rho_{k-1}$ for $k > 2$

(c.) Show that $\rho_2 = 0.7\rho_1 + 0.3\frac{\sigma_e^2}{\gamma_0}$

Question.4 **(2+4+4=10-Points)**

Consider an $AR(2)$ model: $Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + e_t$

(a.) Write down the autoregressive characteristic equation

(b.) What are the conditions required to have stationarity for this model

(c.) If $\phi_1 = 2$, $\phi_2 = -1$, identify the model as a specific ARIMA model. Write down $p, d$,and $q$ and all parameters (the $\phi$'s and $\theta$'s)

Question 5. (**7+5=12-Points**)

Suppose that $\{Y_t\}$ is generated according to: $Y_t = e_t + ce_{t-1} + ce_{t-2} + \ldots + ce_0$, $t > 0$

(a) Find the mean and covariance functions of $\{Y_t\}$. Is $\{Y_t\}$ stationary?

(b) If $W_t = \nabla Y_t$, simplify $W_t$. Is $\{W_t\}$ stationary?

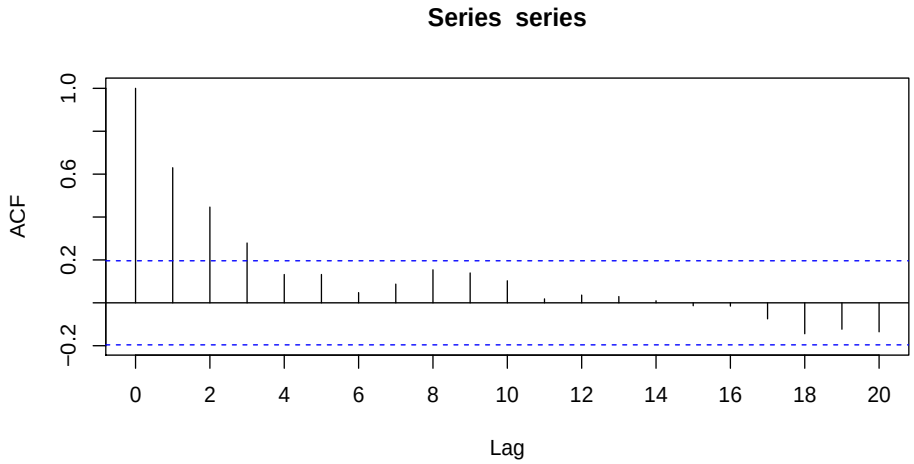
Question 6. (**7+4=11-Points**)

(a) A time series with length 40 produced the sample partial autocorrelation function (PACF) of $\phi_{11} = 0.88$, $\phi_{22} = -0.65$, $\phi_{33} = 0.35$, $\phi_{44} = -0.06$, and $\phi_{55} = 0.00$. Based on this information alone, what model would be tentatively specify for this series?

(b) It is known that the difference of any stationary time series is stationary. However, overdifferencing will create some problems. Mention two problems of overdifferencing

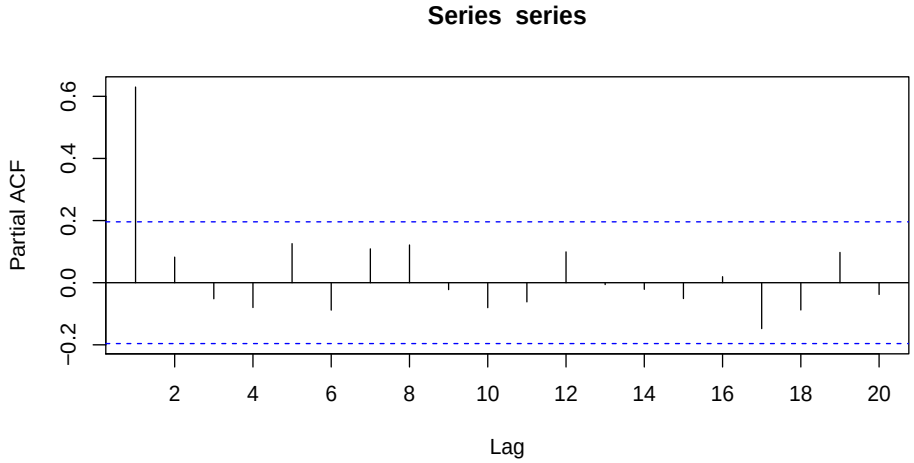
Question 7. **(6 $X$ 2 =12-Points)**

Consider the following graphs of some time series data. For each graph, write a comment and suggest a suitable model if possible.



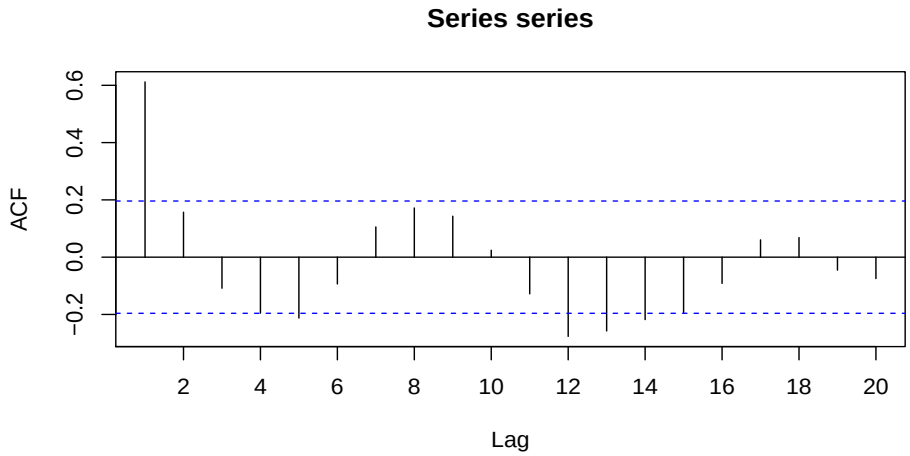

(a)

**Comment:**



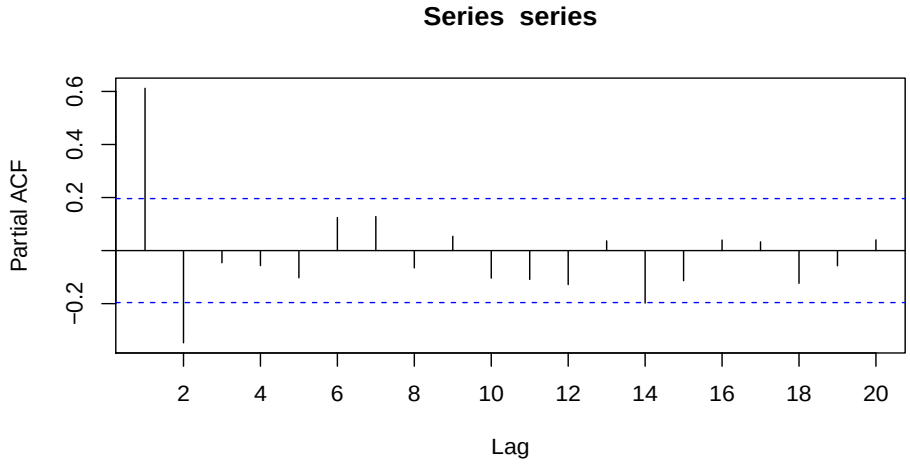

(b)

**Comment:**



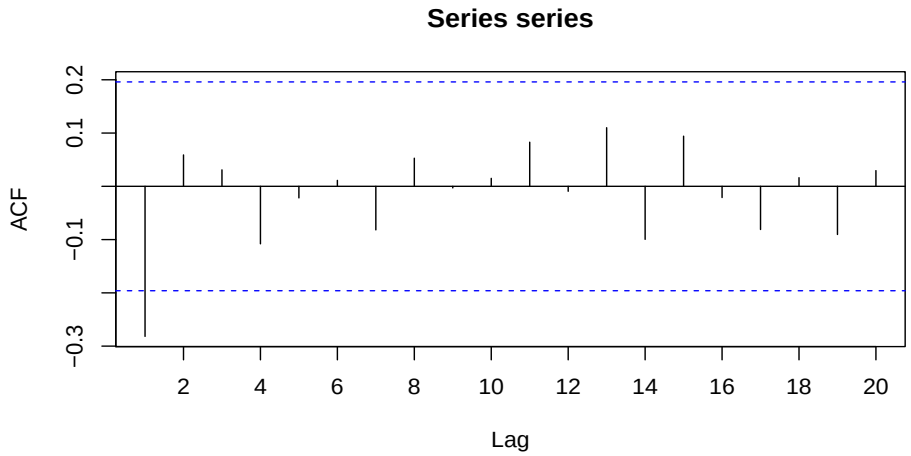

(c)

**Comment:**



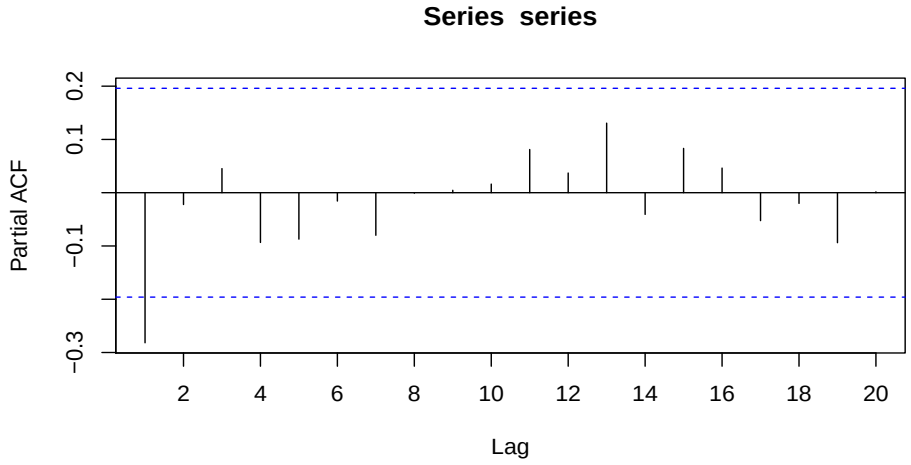

(d)

**Comment:**




(E)

**Comment:**




(F)

**Comment:**

Question 8. **(6 $X$ 2 =12-Points)**

Assume that you have a data file named as ***exam2*** Write the ***R-commands*** for the following

(a) A plot of time series to see the first shape.

(b) A plot of $Y_t$ against $Y_{t-2}$.

(c) A plot of some values of $\lambda$ of the power transformation using Box and Cox method.

(d) A plot the sample autocorrelation function (ACF) based on 30 lags with 10 lag points appear on the horizontal axis

(e) A plot the sample partial autocorrelation function (PACF) based on 18 lags with 6 lag points appear on the horizontal axis

(f) Simulate $n = 100$ observations from the model: $Y_t = 0.9Y_{t-7} + e_t - 0.7e_{t-7}$