King Fahd University Of Petroleum & Minerals
Department Of Mathematics And Statistics
STAT460 : Time Series (161)
First Exam **Wednesday October 30, 2016**
Name: **ID:**

Question Number	Full Mark	Marks Obtained
One	8	
Two	12	
Three	22	
Four	9	
Five	24	
Total	75	

Question.1 (**2+2+2+2=8-Points**) Define the following:

(a) Strictly Stationary process $\{Y_t\}$:

(b) Weakly or Second-order Stationary process $\{Y_t\}$

(c) White noise Time Series:

(d) Random Walk time Series:

Question .2 **(3+6+3=12-Points)**

Let $\{Y_t\}$ be process given by: $Y_t = e_t + 3e_{t-1}$, for $t = 0, 1, 2, \ldots$, and e_t has a mean 0 and variance σ_e^2.

(a) Find the mean and the variance of this time series.

(b) Find the autocovariance function.

(c) Find the autocorrelation function

Question.3 **(4+4+4+10=22-Points)**

Let $\{Y_t\}$ be a random walk with drift μ time series given by: $Y_t = \mu + Y_{t-1} + e_t$, where $Y_0 = 0$.

(a.) Show that $\{Y_t\}$ can be written as $Y_t = t\mu + \sum_{i=1}^{t} e_i$

(b.) Find the the mean of this time series. Is it stationary? Why?

(c.) Find the the autocorrelation function of this time series.

(d.) Let $X_t = Y_t - Y_{t-1}$. Determine wether X_t is stationary. Explain by considering the mean and the autocorrelation functions of X_t.

Question.4 **(6+3=9-Points)**

Suppose that $Y_t = \mu + e_t + e_{t-1}$, where e_t is a white nose with mean 0 and variance σ_e^2. Define $\bar{Y} = \frac{1}{n}\sum_{t=1}^{n} Y_t$.

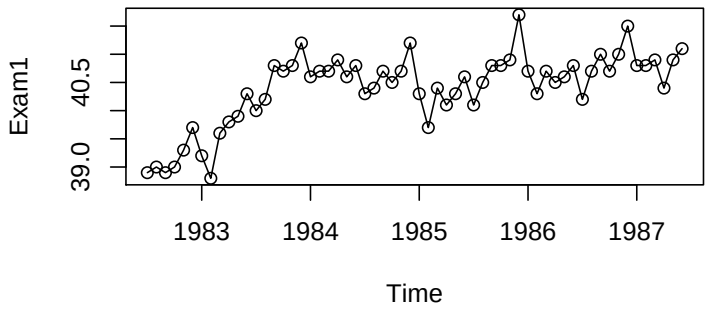
(a) Find $Var(\bar{Y})$

(b) If $Z_t = \mu + e_t$ and $Var(\bar{Z}) = \frac{\sigma_e^2}{n}$. What you would say about the $Var(\bar{Y})$ compared with $Var(\bar{Z})$.

Question 5. **(3 X 8 =24-Points)**

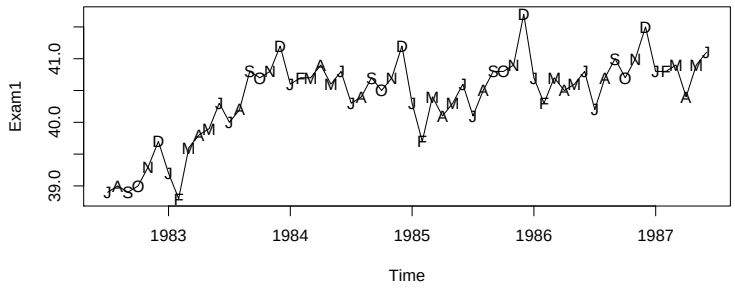
The following output and graphs are based on data example named as exam1data. Write comment after each plot.

(a) The plot of exam1data against time.



Comment:

(b) The plot of exam1data against time using separate symbols for various months.



Comment:

(c) The following is the output of fitting a quadratic trend using LSE.

```
Call: lm(formula = exam1data ~ time(exam1data) + I(time(exam1data)^2))
Residuals: Min       1Q   Median       3Q      Max
-1.00603 -0.25431 -0.02267  0.22884  0.98358

Coefficients:
                           Estimate Std. Error t value Pr(>|t|)
     (Intercept)          -5.122e+05  1.155e+05  -4.433 4.28e-05 ***
     time(exam1data)       5.159e+02  1.164e+02   4.431 4.31e-05 ***
     I(time(exam1data)^2) -1.299e-01  2.933e-02  -4.428 4.35e-05 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 0.423 on 57 degrees of freedom
Multiple R-squared:  0.5921,    Adjusted R-squared:  0.5778
F-statistic: 41.37 on 2 and 57 DF,  p-value: 7.97e-12
```

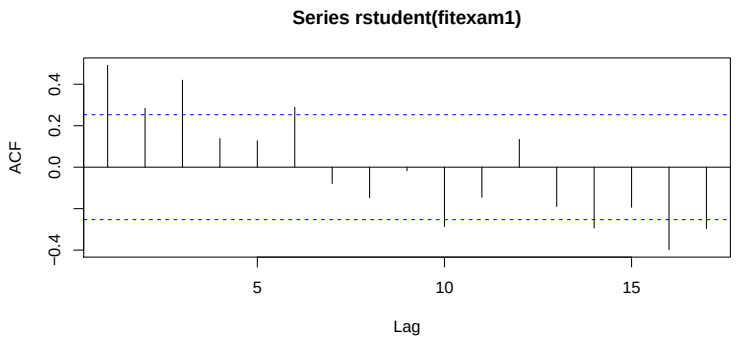
Comment:

(d) The run test is given below:

```
> runs(rstudent(fitexam1))
$pvalue
[1] 0.00012

$observed.runs
[1] 16

$expected.runs
[1] 30.96667

$n1
[1] 31

$n2
[1] 29

$k
[1] 0
```

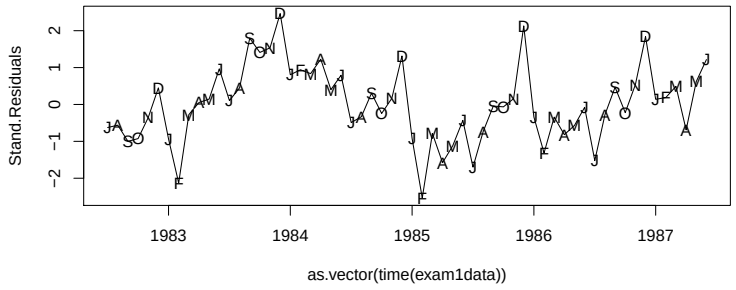
Comment:

(e) The plot of the sample autocorrelations for the standardized residuals .



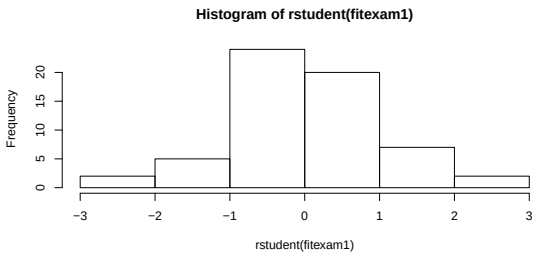
Comment:

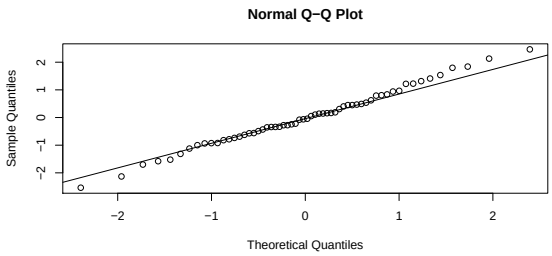
(f) The plot of exam1data against time using separate symbols for various months.



Comment:

(g) The histogram and normal probability plots for the standardized residuals .





Comment:

(h) The Shapiro test.

```
Shapiro-Wilk normality test

data:  rstudent(fitexam1)
W = 0.99385, p-value = 0.9909
```

Comment: