King Fahd University of Petroleum & Minerals
Department of Mathematics and Statistics
MATH 321-01(Term 161)
Final Exam
January 19, 2017

NAME: .....

ID #: . . . . . .

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Calculator is allowed

Question	Points	Score
1	18	
2	18	
3	18	
4	18	
5	10	
6	14	
7	18	
8	18	
9	18	
10	25	
Total	175	

Q1. Given a polynomial of degree 4 as follows:

$$P(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4$$

(i) Suggest an efficient way to evaluate $P(x)$ with only 4 multiplications.

(ii) Assume that coefficients $a_i's$ are in array $a(\ )$, write an algorithm to achieve your suggestion

Q2. (i) State the definition of the degree of precision of a quadrature formula

(ii) Find the constants c_0, c_1, and x_1 so that the quadrature formula

$$\int_0^1 f(x)dx = c_0 f(0) + c_1 f(x_1)$$

has the highest possible degree of precision. What is that degree?

Q3. Factor the matrix A into $A = LU$, where L is lower triangular with 1s on its diagonal and U is upper triangular

$$A = \begin{bmatrix} 2 & 3 & -1 \\ 4 & 4 & -1 \\ -2 & -3 & 4 \end{bmatrix}$$

then solve the liner system $Ax = b$ with $b = [2, -1, 1]^T$

Q4. A natural cubic spline S on $[0, 2]$ is defined by

$$s(x) = \begin{cases} s_0(x) = 2 - x - x^2 + 3x^3 & 0 \le x \le 1 \\ s_1(x) = a + b(x-1) + c(x-1)^2 + d(x-1)^3 & 1 \le x \le 2 \end{cases}$$

Find a, b, c, and d.

Q5. Let $f \in C^2[x_0, x_0 + h]$ with $h > 0$. Show how to derive an approximation to $f'(x_0)$, with error term.

Q6. Consider $f \in C^2[a,b]$, $h = \frac{b-a}{n}$, $x_j = a + jh$, $j = 0, 1, \ldots, n$ and $\xi \in (a,b)$. The error in the composite Trapezoidal rule is

$$-\frac{b-a}{12}h^2 f''(\xi)$$

. Determine the values of n and h required to approximate

$$\int_0^2 xe^x dx$$

to within 10^{-4} by the composite Trapezoidal rule.

Q7. Consider the function $f(x) = e^x - 5 + 5x$

(i) Construct a fixed point iteration for finding a root of $f(x)$.

(ii) Check whether it has the necessary property for convergence at $x = 0$.

(iii) Perform two iterations starting at $x = 0$

Q8. For a function f, the forward divided differences are given by

$$\begin{array}{llll} x_0 = 0.0 & f[x_0] = ... & & \\ x_1 = 0.4 & f[x_1] = ... & f[x_0, x_1] = ... & \\ x_2 = 0.7 & f[x_2] = 6 & f[x_1, x_2] = 10 & f[x_0, x_1, x_2] = \frac{50}{7} \end{array}$$

(i) Determine the missing entries in the table

(ii) Use the divided differences coefficients to find the interpolating polynomial $P_2(x)$.

Q9. Given $3y' + \sqrt{y} = e^{0.1x}$, with $y(0.3) = 5$. Using a step size of $h = 0.3$, find $y(0.9)$ using Euler's method then find the best estimate of $\frac{dy}{dx}(0.9)$

Q10. Describe **briefly** the following:

(i) The two main types of errors in numerical computing

(ii) Absolute and Relative Error.

(iii) The difference between Direct and Iterative Methods for solving linear systems of equations

(iv) The most convenient procedure for fitting the best line to a collection of data $\{(x_i, y_i)\}_{i=1}^{m}$

(v) Let $f \in [a, b]$. How many points would you use to construct a Lagrange polynomial of degree 10?