King Fahd University of Petroleum & Minerals
Department of Mathematics and Statistics
MATH 321-01(Term 161)
Exam II
December 20, 2016

NAME: ....

ID #: . . . .

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Question	Points	Score
1	20	
2	10	
3	20	
4	15	
5	10	
6	15	
7	10	
Total	100	

Q1. Consider the following ordinary differential equation

$$3\frac{dy}{dx} + 5y^2 = \sin x, \quad y(0.3) = 5$$

(a) rewrite the equation so you can use Euler's method to solve it.

(b) using a step size $h = 0.3$, find the value of $y(0.9)$ using Euler's method.

Q2. Find the degree of accuracy of the following integration formula

$$\int_0^a f(x)dx = \frac{a}{2}f(0) + \frac{a}{2}f(a)$$

where $a > 0$

Q3. Compare the Trapezoidal rule and Simpson's rule approximations to

$$\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \sin^2 x \, dx$$

and find a bound for the theoretical error in both cases.

Trapezoidal Rule: $\int_a^b f(x)dx = \frac{h}{2}[f(x_0) + f(x_1)] - \frac{h^3}{12} f''(\xi)$

Simpsons's Rule: $\int_a^b f(x)dx = \frac{h}{3}[f(x_0) + 4f(x_1) + f(x_2)] - \frac{h^5}{90} f^{(4)}(\xi)$

Q4. Given a function f defined on $[a, b]$ and a set of nodes $a = x_0 < x_1 < x_2 = b$. State all the conditions that need to be satisfied so that S will be a cubic spline interpolant for f.

Q5. Let $D = \{(t, y) \mid a \leq t \leq b \text{ and } -\infty < y < \infty\}$ and that $f(t, y)$ is continuous on D. State and define the condition that need to be satisfied by f such that the initial-value problem

$$y'(t) = f(t, y), \quad a \leq t \leq b, \quad y(a) = \alpha,$$

has a unique solution $y(t)$ for $a \leq t \leq b$.

Q6. Use Gaussian elimination with partial pivoting to solve the following linear system

$$
\begin{aligned}
x_1 - x_2 + x_3 &= 5 \\
7x_1 + 5x_2 - x_3 &= 8 \\
2x_1 + x_2 + x_3 &= 7
\end{aligned}
$$

Q7. Consider he following numerical differentiation formulas

$$\boxed{f'(x_0) = \frac{1}{2h}[-3f(x_0) + 4f(x_0 + h) - f(x_0 + 2h)] - \frac{h^2}{3} f^{(3)}(\xi_0)}$$

$$\boxed{f'(x_0) = \frac{1}{2h}[f(x_0 + h) - f(x_0 - h)] - \frac{h^2}{6} f^{(3)}(\xi_1)}$$

(a) Discuss the difference between these two formulas and when it is useful to use each one.

(b) Explain why numerical differentiation is unstable.