King Fahd University of Petroleum & Minerals
Department of Mathematics and Statistics

MATH 321-01(Term 161)

Exam I

October 27, 2016

NAME: ....

ID #: . . . .

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Calculator is allowed

Question	Points	Score
1	15	
2	15	
3	15	
4	10	
5	10	
6	20	
7	15	
Total	100	

Q1. (a) Let $f \in C^2[a, b]$. Show how to derive Newton's method using Taylor polynomial for $f(x)$ expanded about p_0.

(b) Show how the Secant method is derived from Newton's method.

(c) Show the the iterative method

$$p_n = \frac{f(p_{n-1})p_{n-2} - f(p_{n-2})p_{n-1}}{f(p_{n-1}) - f(p_{n-2})}$$

is mathematically equivalent to the Secant method.

Q2. Consider the following data points:

x	0	2	3	4
f(x)	7	11	28	63

a) Find the second Lagrange interpolating polynomial for $f(x)$ using the first 3 nodes.

b) Use divided differences to find the interpolating polynomial $P(x)$ of all of the above data.

Q3. Let $f(x) = e^x - 3x^2$. Find the zero of $f(x)$ in the interval $[3, 5]$ using:

(a) Bisection method

(b) Newton's method with $p_0 = 3$

(c) Secant method with $p_0 = 3$, $p_1 = 4$

Carry out the first two iterations in each case.

Q4. Show how to reduce the number of multiplication needed to evaluate the following polynomial. Justify your answer.

$$P(x) = 3x^4 + 4x^3 + 5x^2 - 5x + 1$$

Q5. Explain why Taylor polynomials are not used in function approximation.

Q6. The quadratic equation $x^2 + x - 1 = 0$ has a positive root $x^* = \frac{-1+\sqrt{5}}{2} \approx 0.618$. The equation can be rearranged to yield two equivalent fixed-point equations:

$$x = g_1(x) = 1 - x^2 \qquad \text{and} \quad x = g_2(x) = \sqrt{1 - x}$$

For which function (g_1 or g_2) do the fixed-point iterates converge when p_0 is sufficiently near x^*. Why?

Q7. Show that $g(x) = \frac{x^2+3}{5}$ has a unique fixed-point on $[0, 1]$ and estimate how many iterations n are required to obtain an absolute error less than 10^{-4} when $p_0 = 1$.