King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math 280
Final Exam, First Semester (161), 2016
Net Time Allowed: 180 minutes

Name:———————————————

ID:——————Section:————Serial:————

Q#	Marks	Maximum Marks
1		5
2		15
3		5
4		6
5		10
6		15
7		10
8		15
9		10
10		10
11		10
12		10
13		10
Total		130

1. Write clearly.
2. Show all your steps.
3. No credit will be given to wrong steps.
4. Do not do messy work.
5. Calculators and mobile phones are NOT allowed in this exam.
6. Turn off your mobile.

1. (5 points) If A and B are row equivalent matrices and A is invertible then B is invertible.

2. (15 points) Consider the vector space $M_{2\times 2}$ of real 2×2 matrices. Let V_1 be the set of matrices of the form $\begin{bmatrix} a & b \\ -a & c \end{bmatrix}$ and V_2 be the set of matrices of the form $\begin{bmatrix} p & -p \\ q & r \end{bmatrix}$.

(a) Prove $V_1 \cap V_2$ are subspace of $M_{2\times 2}$.

(b) Find bases for $V_1 \cap V_2$. Give the dimensions of these spaces.

3. (5 points) Show that the following transformation $T : \mathbb{R}^3 \to \mathbb{R}^2$ is not linear

$$T(x, y, z) = (xy, z)$$

4. (6 points) Find the $1-$, $2-$, and ∞-norms of $\begin{bmatrix} 1+i \\ 1-i \\ 1 \\ 4i \end{bmatrix}$

5. (10 points) Show that $\text{Trace}(A^T B)^2 \leq \text{Trace}(A^T A)\text{Trace}(B^T B)$, for all $A, B \in \mathbb{R}^{m \times n}$

6. (15 points) Given the matrix $A = \begin{bmatrix} 0 & -2 & 1 \\ 2 & 4 & -1 \\ 2 & 1 & 2 \end{bmatrix}$

(a) Diagonalize A.

(b) Use the result obtained in (a) state how to find A^n.

(c) State how to find $\exp(A)$

7. A matrix A is said to be nilpotent if $A^k = 0$ for some integer k. Prove that if A is nilpotent then 0 is the only eigenvalue of A.

8. (15 points) Let $A = \begin{bmatrix} 1 & -1 & -1 \\ 1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix}$

(a) Find orthonormal basis for the column space of the matrix A.

(b) Find the QR-factorization of A by using part(a).

9. (10 points) For the space P_3 of polynomials, determine the change of basis matrix from $\mathcal{B}_1$ to $\mathcal{B}_2$, where

$$\mathcal{B}_1 = \{1, x, x^2\} \text{ and } \mathcal{B}_2 = \{1, 1+x, 1+x+x^2\},$$

and then find the coordinates of $q(x) = 3 + 2x + 4x^2$ relative to $\mathcal{B}_2$.

10. Using the trace inner product, determine the angle between the identity matrix $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

11. If $\{u_1, u_2, \cdots, u_n\}$ is an orthonormal basis for an inner-product space V, show that

$$\langle x, y\rangle = \sum_{i=1}^{n} \langle x, u_i\rangle\langle u_i, y\rangle$$

holds for every $x, y \in V$.

12. (10 points) Given the quadratic equation

$$x^2 + 4xy + y^2 + 3x + y - 1 = 0$$

find a change of cordinates so that the resulting equation represents a conic in standard position.

13. (10 points) The function $f(x,y) = (x^2 - 2x)\cos y$ has a critical point at $(1,\pi)$. Determine wether the given stationary point is local maximum, minimum or saddle point.