King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math 280
Major Exam II, Semester I, 2016-2017
Net Time Allowed: 120 minutes

Name:————————————————
ID:————————Section:————Serial:————————

Q#	Marks	Maximum Marks
1		7
2		8
3		10
4		10
5		10
6		15
7		10
8		15
9		15
Total		100

1. Write clearly.
2. Show all your steps.
3. No credit will be given to wrong steps.
4. Do not do messy work.
5. Calculators and mobile phones are NOT allowed in this exam.
6. Turn off your mobile.

1. Let V be the set of all ordered pairs of real numbers. Define scalar multiplication and addition on V by

$$\alpha(x_1, x_2) = (\alpha x_1, \alpha x_2)$$
$$(x_1, x_2) \oplus (y_1, y_2) = (x_1 + y_1, 0)$$

Is V a vector space with these operations? Justify your answer.

2. Let S be a subset of the vector space $\mathbb{R}^{n\times n}$, where $S = \{A : A \text{ is } n\times n \text{ matrix and Trace}(A) = 0\}$. Is S a subspace of $\mathbb{R}^{n\times n}$? Explain?

3. Determine whether or not the following sets span the same subspace:

$$\mathcal{A} = \left\{ \begin{bmatrix} 1 \\ 2 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 6 \\ 1 \\ 4 \end{bmatrix} \right\}, \quad \mathcal{B} = \left\{ \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \right\}$$

4. Show that if A is similar to B, then $\mathrm{tr}(A) = \mathrm{tr}(B)$.

5. Do the following set of matrices form a basis for $\mathbb{R}^{2\times 2}$

$$\left\{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \right\}$$

6. Let $A = \begin{bmatrix} 1 & 3 & -4 \\ 2 & -1 & -1 \\ -1 & -3 & 4 \end{bmatrix}$

(a) Find a basis for the row space of A. What is the dimension of the row space?

(b) Find a basis for the column space of A. What is the rank of A?

(c) Find a basis for the null space of A. What is the nullity of A?

7. Let E and F be two ordered bases for $\mathbb{R}^2$, where $E = \left\{ v_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, v_2 = \begin{pmatrix} 1 \\ 5 \end{pmatrix} \right\}$ and $F = \left\{ v_1 = \begin{pmatrix} 1 \\ -3 \end{pmatrix}, v_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right\}$. Find the transition matrix from E to F and use it to find the coordinates of $x = 2v_1 - 3v_2$.

8. Let L be the operator on P_3 defined by

$$L(p(x)) = xp'(x) + p''(x)$$

(a) Find the matrix A representing L with respect to $[1, x, x^2]$.

(b) Find the matrix B representing L with respect to $[1, x, 1 + x^2]$.

(c) Find the matrix S such that $B = S^{-1}AS$.

(d) If $p(x) = a_0 + a_1x + a_2(1 + x^2)$. calculate $L^n(p(x))$.

9. Let T be a a mapping from $\mathbb{R}^3$ onto $\mathbb{R}^3$ defined by

$$T\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 \\ x_2 - 2x_3 \\ x_1 + 2x_3 \end{pmatrix}$$

(a) Show that T is a linear transformation.

(b) Find the kernel of T.

(c) Find the image of the subspace $S = \{(a, -a, a)^T : a \text{ is a real number}\}$.