King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math 280
Major Exam I, Semester I, 2016-2017
Net Time Allowed: 120 minutes

Name:____

ID:____________________ Section:__________ Serial:______________

Q#	Marks	Maximum Marks
1		10
2		10
3		10
4		15
5		10
6		10
7		10
8		10
9		15
Total		100

1. Write clearly.
2. Show all your steps.
3. No credit will be given to wrong steps.
4. Do not do messy work.
5. Calculators and mobile phones are NOT allowed in this exam.
6. Turn off your mobile.

1. Find the reduced row echelon form for

$$\begin{bmatrix} 1 & 2 & -1 & 1 & 0 & 5 \\ 1 & 1 & 2 & 2 & 1 & 2 \\ 1 & -1 & -1 & 3 & 4 & -3 \end{bmatrix}$$

2. For matrices $A_{r\times r}, B_{s\times s}$, and $C_{r\times s}$ such that A and B are nonsingular, show that

$$\begin{bmatrix} A & C \\ 0 & B \end{bmatrix}^{-1} = \begin{bmatrix} A^{-1} & -A^{-1}CB^{-1} \\ 0 & B^{-1} \end{bmatrix}$$

3. Find X in the matrix equation

$$X = XA + B,$$

where $A = \begin{bmatrix} 0 & -2 & -1 \\ 0 & 0 & -3 \\ 0 & 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \end{bmatrix}$

4. Let $A = \begin{bmatrix} 1 & -1 & -2 \\ 2 & -3 & -5 \\ 1 & 3 & 5 \end{bmatrix}$

(a) Find elementary matrices E_1, E_2, E_3 such that

$$E_3 E_2 E_1 A = U.$$

(b) Using (a) compute the LU factorization of the matrix A

(c) Using (b), solve the system $Ax = b$, where $b = (1, 3, -3)^T$.

5. Using Cramer's rule, find only the value of the angle β such that

$$\begin{aligned} 2\sin\alpha - \cos\beta + 3\tan\gamma &= 3, \\ 4\sin\alpha + 2\cos\beta - 2\tan\gamma &= 2, \\ 6\sin\alpha - 3\cos\beta + \tan\gamma &= 9. \end{aligned}$$

where $0 \leq \beta \leq 2\pi$.

6. Let A and B be symetric matrices of the same size. Prove that AB is symetric if and only if $AB = BA$.

7. A square matrix is said to be idempotent if $A^2 = A$. Show that if A and B are idempotent and $AB = BA$, then AB is idempotent.

8. Solve the equation

$$\begin{vmatrix} 1-x & 7+2x & 3x \\ 4+2x & 10-4x & 6-6x \\ 2 & 4 & 5 \end{vmatrix} = 0$$

9. Compute the following

   (a) Prove that $[\text{adj}(A)]^{-1} = \text{adj}(A^{-1})$

   (b) Find $[\text{adj}(A)]^{-1}$ given $A^{-1} = \begin{bmatrix} 3 & 0 & 2 \\ 2 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix}$