KFUPM/ Department of Mathematics & Statistics/ 161/ MATH 260-06/ Quiz 3

Name: **ID #:** **Serial #:**

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1. Find the eigenvalues and corresponding eigenvectors of the matrix $A = \begin{bmatrix} 0 & -12 \\ 3 & 0 \end{bmatrix}$.

Sol.

- Eigenvalues: $|A - \lambda I| = \lambda^2 + 36$, so the eigenvalues are $\lambda_1 = 6i$ and $\lambda_2 = -6i$.
- Eigenvectors: $A - \lambda_1 I = \begin{bmatrix} -6i & -12 \\ 3 & -6i \end{bmatrix} \longrightarrow \begin{bmatrix} i & 2 \\ 0 & 0 \end{bmatrix}$. Hence an eigenvector for λ_1 is $v_1 = (2, -i)$.

  Since $\lambda_2 = \overline{\lambda_1}$ (the conjugate of λ_1), an eigenvector for λ_2 is $v_2 = \overline{v_1} = (2, i)$.

2. Is the matrix $A = \begin{bmatrix} 3 & -3 & 1 \\ 2 & -2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ diagonalizable? If it is, find a diagonalizing matrix P and diagonal matrix D such that $A = PDP^{-1}$.

Sol.

- Eigenvalues: $|A - \lambda I| = \begin{vmatrix} 3-\lambda & -3 & 1 \\ 2 & -2-\lambda & 1 \\ 0 & 0 & 1-\lambda \end{vmatrix} = -\lambda(\lambda - 1)^2$. So the eigenvalues are $\lambda_1 = 1$ (multiplicity 2) and $\lambda_2 = 0$.
- Eigenvectors (x, y, z): $A - \lambda_1 I = \begin{bmatrix} 2 & -3 & 1 \\ 2 & -3 & 1 \\ 0 & 0 & 0 \end{bmatrix} \longrightarrow \begin{bmatrix} 2 & -3 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, with 2 free variables y, z. Since multiplicity of λ_1 is also 2, the matrix A is diagonalizable.
  - For λ_1, eigenvector (x, y, z) can be written $(3y/2 - z/2, y, z) = y(3/2, 1, 0) + z(-1/2, 0, 1)$. Hence a basis for the eigenspace of λ_1 is $\{(3, 2, 0), \ (-1, 0, 2)\}$.
  - For λ_2, $A - \lambda_2 I = \begin{bmatrix} 3 & -3 & 1 \\ 2 & -2 & 1 \\ 0 & 0 & 1 \end{bmatrix} \longrightarrow \begin{bmatrix} 2 & -2 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$, with 1 free variable y. In this case, eigenvector (x, y, z) can be written $(y, y, 0)$. A basis for the eigenspace of λ_2 is $\{(1, 1, 0)\}$.
- The required matrices are: $P = \begin{bmatrix} 3 & -1 & 1 \\ 2 & 0 & 1 \\ 0 & 2 & 0 \end{bmatrix}$ and $D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.