KFUPM/ Department of Mathematics & Statistics/ 161/ MATH 260-05/ Quiz 3

Name: **ID #:** **Serial #:**

---

1. Find the eigenvalues and corresponding eigenvectors of the matrix $A = \begin{bmatrix} 0 & 2 \\ -18 & 0 \end{bmatrix}$.

   Sol.

   - Eigenvalues: $|A - \lambda I| = \lambda^2 + 36$, so the eigenvalues are $\lambda_1 = 6i$ and $\lambda_2 = -6i$.
   - Eigenvectors: $A - \lambda_1 I = \begin{bmatrix} -6i & 2 \\ -18 & -6i \end{bmatrix} \longrightarrow \begin{bmatrix} -3i & 1 \\ 0 & 0 \end{bmatrix}$. Hence an eigenvector for λ_1 is $v_1 = (1, 3i)$.

     Since $\lambda_2 = \overline{\lambda_1}$ (the conjugate of λ_1), an eigenvector for λ_2 is $v_2 = \overline{v_1} = (1, -3i)$.

2. Is the matrix $A = \begin{bmatrix} 3 & -2 & 0 \\ 0 & 1 & 0 \\ -4 & 4 & 1 \end{bmatrix}$ diagonalizable? If it is, find a diagonalizing matrix P and diagonal matrix D such that $A = PDP^{-1}$.

   Sol.

   - Eigenvalues: $|A - \lambda I| = \begin{vmatrix} 3-\lambda & -2 & 0 \\ 0 & 1-\lambda & 0 \\ -4 & 4 & 1-\lambda \end{vmatrix} = (1-\lambda)(3-\lambda)(1-\lambda)$. So the eigenvalues are $\lambda_1 = 1$ (multiplicity 2) and $\lambda_2 = 3$.
   - Eigenvectors (x, y, z): $A - \lambda_1 I = \begin{bmatrix} 2 & -2 & 0 \\ 0 & 0 & 0 \\ -4 & 4 & 0 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, with 2 free variables y, z. Since multiplicity of λ_1 is also 2, the matrix A is diagonalizable.
     - For λ_1, eigenvector (x, y, z) can be written $(y, y, z) = y(1,1,0) + z(0,0,1)$. Hence a basis for the eigenspace of λ_1 is $\{(1,1,0),\ (0,0,1)\}$.
     - For λ_2, $A - \lambda_2 I = \begin{bmatrix} 0 & -2 & 0 \\ 0 & -2 & 0 \\ -4 & 4 & -2 \end{bmatrix} \longrightarrow \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ -2 & 2 & -1 \end{bmatrix}$, with 1 free variable z. In this case, eigenvector (x, y, z) can be written $(-z/2, 0, z)$. A basis for the eigenspace of λ_2 is $\{(-1, 0, 2)\}$.
   - The required matrices are: $P = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & 1 & 2 \end{bmatrix}$ and $D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.