Dept. Math & Stat. Math 232

Quiz N° 1

EX1: Explain why the following argument is valid:

If roses are raid, then sugar is sweet
Roses are red,
Therefore sugar is sweet

Solution. The propositional form of this argument is

$$\begin{array}{l} P \longrightarrow Q \\ P \\ \hline \therefore Q \end{array}$$

This is Modus Ponens.
The proof of validity of this argument is as follows:

$$\left[(P \longrightarrow Q) \wedge P\right] \longrightarrow Q$$

$$\equiv \overline{(\bar{P} \vee Q) \wedge P} \vee Q \equiv (\overline{\bar{P}} \wedge \bar{Q}) \vee \bar{P} \vee Q$$
$$\equiv \overline{\bar{P} \vee Q} \vee (\bar{P} \vee Q)$$
$$\equiv T$$

EX2. Let $P(x)$: " 3^x+4 is prime" and $S=\{0,1,2,3,\dots\}$.

① $P(0)$: " $3^0+4=5$ is prime" is a true statement.
$P(1)$: " $3^1+4=7$ is prime" " " "
$P(2)$: " $3^2+4=13$ is prime" " " "
$P(3)$: " $3^3+4=31$ is prime" " " "
$P(4)$: $3^4+4=85$ is prime" is a false statement

So the quantified statement
"$\forall x \in S, P(x)$" is false.

(2) The quantified statement "$\exists x \in S, P(x)$" is true as $P(a)$ is true.

EX 3.

Use logical equivalences to show that for P, Q statements

$$\overline{P \vee (\overline{P \wedge Q})}$$

is a contradiction.

Solution.

$$\overline{P \vee (\overline{P \wedge Q})} \equiv \overline{P} \wedge (\overline{\overline{P \wedge Q}})$$
$$\equiv \overline{P} \wedge (P \wedge Q)$$
$$\equiv (\overline{P} \wedge P) \wedge Q$$
$$\equiv C \wedge Q$$
$$\equiv C$$

EX 4. Let P, Q be statements. We let with $P \oplus Q$ be the statement $(P \vee Q) \wedge \overline{P \wedge Q}$.

(1) Truth table of $P \oplus Q$.

(2) Use logical equivalence to show that

$$\overline{P \oplus Q} \equiv P \leftrightarrow Q$$
$$\equiv (P \vee Q) \rightarrow (P \wedge Q).$$

Solution.

(1) $P \oplus Q \equiv (P \vee Q) \wedge \overline{P \wedge Q}$

P	Q	$P \vee Q$	$P \wedge Q$	$\overline{P \wedge Q}$	$P \oplus Q$
T	T	T	T	F	F
T	F	T	F	T	T
F	T	T	F	T	T
F	F	F	F	T	F

(2) $\overline{P \oplus Q} \equiv \overline{(P \vee Q) \wedge (\overline{P \wedge Q})} \equiv \overline{(P \vee Q)} \vee (P \wedge Q)$

$\equiv (P \vee Q) \rightarrow (P \wedge Q)$

So $\overline{P \oplus Q} \equiv (P \vee Q) \rightarrow (P \wedge Q)$

Also, we have

$$(P \vee Q) \to (P \wedge Q) \equiv \overline{P \vee Q} \vee (P \wedge Q)$$

$$\equiv (\bar{P} \wedge \bar{Q}) \vee (P \wedge Q)$$

$$\equiv \left(\bar{P} \vee (P \wedge Q)\right) \wedge \left(\bar{Q} \vee (P \wedge Q)\right)$$

$$\equiv (\bar{P} \vee P) \wedge (\bar{P} \vee Q) \wedge (\bar{Q} \vee P) \wedge (\bar{Q} \vee Q)$$

$$\equiv T \wedge (\bar{P} \wedge Q) \wedge (\bar{Q} \vee P) \wedge T$$

$$\equiv (\bar{P} \vee Q) \wedge (\bar{Q} \vee P)$$

$$\equiv (P \to Q) \wedge (Q \to P)$$

$$\equiv P \leftrightarrow Q$$

EX5.

Use logical equivalences to show that

$$[(P \to Q) \wedge (Q \to R)] \to (P \to R)$$

is a tautology.

Solution.

$$[(P \to Q) \wedge (Q \to R)] \to (P \to R)$$

$$\equiv \overline{(\bar{P} \vee Q) \wedge (\bar{Q} \vee R)} \vee (\bar{P} \vee R)$$

$$\equiv (P \wedge \bar{Q}) \vee (Q \wedge \bar{R}) \vee \bar{P} \vee R$$

$$\equiv [(P \wedge \bar{Q}) \vee \bar{P}] \vee [(Q \wedge \bar{R}) \vee R]$$

$$\equiv [(P \vee \bar{P}) \wedge (\bar{Q} \vee \bar{P})] \vee [(Q \vee R) \wedge (\bar{R} \vee R)]$$

$$\equiv [T \wedge (\bar{Q} \vee \bar{P})] \vee [(Q \vee R) \wedge T]$$

$$\equiv \bar{Q} \vee \bar{P} \vee Q \vee R \equiv (\bar{Q} \vee Q) \vee \bar{P} \vee R \equiv T \vee \bar{P} \vee R$$

$$\equiv T$$