KFUPM, DEPARTMENT OF MATHEMATICS AND STATISTICS

MATH 232: EXAM II, T 161, DECEMBER 18, 2016

17:30–19:30 PM

Name : ....

ID : ....

Exercise 1 (10 pts)**.** Let φ be the positive root of the quadratic equation

$$x^2 - x - 1 = 0.$$

Let $(F_n, n \in \mathbb{N})$ be the sequence defined recursively by $F_0 = 1, F_1 = 1$, and $F_n = F_{n-1} + F_{n-2}$, for $n > 1$.

Show that $F_n \leq \varphi^n$, for each integer $n \geq 0$.

Exercise 2 (10 pts). Let $(u_n, n \in \mathbb{N})$ be the sequence defined recursively by $u_0 = 0, u_1 = 1$ $u_{n+1} = u_n + 2u_{n-1}$, for $n \geq 0$.

(1) Show that $u_n \in \mathbb{N}$, for each $n \in \mathbb{N}$.

(2) Show that $u_n = \dfrac{1}{3}(2^n - (-1)^n)$, for each $n \in \mathbb{N}$.

Exercise 3 (10 pts)**.** Let $(u_n, n \in \mathbb{N})$ be the sequence defined by $u_0 = 1$, $u_1 = 2$ and $u_{n+2} = 5u_{n+1} - 6u_n$, for $n \geq 0$.

(1) Find u_2, u_3, u_4.

(2) Find a formula for u_n.

Exercise 4 (10 pts)**.**

(1) Show that the interval $[0, 1]$ equipped with the usual order is not well ordered.

(2) Show that the set $\{2^n \mid n \in \mathbb{N}\}$ equipped with the usual order is well ordered.

Exercise 5 (15 pts)**.** Let $\sim$ be the relation defined on $\mathbb{Z}$ by;

$$x \sim y \iff x^2 \equiv y^2 \pmod 5$$

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(1) Show that $\sim$ is an equivalence relation.

(2) Find $\overline{0}, \overline{1}, \overline{2}$.

(3) Find the cardinality of the quotient set $\mathbb{Z}/\sim$.

Exercise 6 (10 pts)**.** Let R be the binary relation defined on $\mathbb{R}^2$ by:

$$(x,y)R(x',y') \text{ if and only if } \mid x' - x \mid \leq y' - y.$$

Show that R is an ordering on $\mathbb{R}^2$ (reflexive, antisymmetric and transitive).

Exercise 7 (15 pts).

(1) Give the table ofmultiplication on $\mathbb{Z}_8$.

(2) Deduce from (1) the solution of the congruence equation:

$$2x \equiv 4 \pmod{8},$$

where the unknown x is in $\mathbb{Z}$.

(3) Deduce from (1) the solution of the congruence equation:

$$x^2 \equiv 4 \pmod{8},$$

where the unknown x is in $\mathbb{Z}$.