KFUPM, DEPARTMENT OF MATHEMATICS AND STATISTICS

MATH 232: EXAM I, SEMESTER (161), NOVEMBER 05, 2016

10:00–12:00 am

Name : ....

ID : ....

Exercise 1 (12 pts)**.**

(1) Explain why $9 \equiv 2 \pmod 7$ and $4 \equiv -3 \pmod 7$.

(2) Show that for each integer $n \geq 0$, $3^{2n+1} + 2^{n+2}$ is divisible by 7.

(3) If today is Saturday, then what day will be after $(3^{2017} + 2^{1010} + 2)$ days?

Exercise 2 (10 pts)**.** Let $x \in \mathbb{R}$. Show that:

(1) If $\left|\dfrac{\sin x}{x^2 - x + 1}\right| > 2$, then $e^x > 2016$.

(2) If $| x - 1 | < 2017$, then $\dfrac{e^{-x^2}}{x^2 - 2x + 5/4} \leq 4$.

Exercise 3 (12 pts)**.** Let $a_1, a_2, b_1, b_2 \in \mathbb{R}$.

(i) Show that if $a_1 \geq a_2$ and $b_1 \geq b_2$, then

$$a_1 b_1 + a_2 b_2 \geq a_1 b_2 + a_2 b_1.$$

(ii) Show that for all real numbers α, β, we have

$$\alpha^2 + \beta^2 \geq 2\alpha\beta.$$

(iii) Use the result of (i) to show that for all real numbers α, β, we have

$$\alpha^2 + \beta^2 \geq 2\alpha\beta.$$

Exercise 4 (8 pts)**.** Show that if n is not divisible by 3, then $n^2 + 2$ is divisible by 3.

Exercise 5 (12 pts)**.** Let $a \in \mathbb{Z}$. Show that if a is a perfect square(i.e., there is $a \in \mathbb{Z}$, such that $a = b^2$), then $a \not\equiv 2 \pmod 4$.

Exercise 6 (8 pts)**.** Let $P_1, P_2, \dots P_n$ and Q be statements. Show that:

$$[(P_1 \vee P_2, \dots \vee P_n) \longrightarrow Q] \equiv [(P_1 \longrightarrow Q) \wedge (P_2 \longrightarrow Q) \wedge \dots \wedge (P_n \longrightarrow Q)].$$

Exercise 7 (38 pts)**.** For P, Q statements, we denote by $P \oplus Q$ the statement $(P \vee Q) \wedge \overline{P \wedge Q}$. Show that the following properties hold:

(1) $P \oplus Q \equiv (P \wedge \overline{Q}) \vee (Q \wedge \overline{P})$.

(2) $\overline{P \oplus Q} \equiv (P \longleftrightarrow Q)$.

(3) $P \oplus Q \equiv \overline{P} \oplus \overline{Q}$.

(4) If C is a contradiction, then $P \oplus C \equiv P$ and $P \oplus P \equiv C$.

(5) $(P \oplus Q) \oplus R \equiv P \oplus (Q \oplus R)$.

(6) $(P \oplus Q) \wedge R \equiv (P \wedge R) \oplus (Q \wedge R)$.

Use the results of the previous questions (5) and (6) to show that, if A, B are subsets of a universal set U, then we have:

(7) $(A\Delta B)\Delta C = A\Delta(B\Delta C)$.

(8) $(A\Delta B) \cap C = (A \cap C)\Delta(B \cap C)$.