King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

MATH 202 - Final Exam - Term 161

Duration: 180 minutes
(7:00-10:00 pm)
January 12, 2017

Name: Key

Section Number: ______ ID Number: ______

Serial Number: ______

Class Time: ______ Instructor's Name: ______

Instructions:

1. Calculators and Mobiles are not allowed.
2. Write neatly.
3. **Show all your work**. No points for answers without justification.
4. Make sure that you have 12 pages of problems (Total of 12 Problems)

Question Number	Points	Maximum Points
1		12
2		12
3		12
4		12
5		12
6		12
7		12
8		12
9		12
10		12
11		10
12		10
Total		140

7-copy

1. Find two linearly independent power series solutions, about $x = 0$, to

$$(x^2+1)\,y'' + xy' - y = 0$$

Assume $y = \sum_{n=0}^{\infty} c_n x^n$

$y' = \sum_{n=1}^{\infty} n c_n x^{n-1}$ and $y'' = \sum_{n=2}^{\infty} n(n-1) c_n x^{n-2}$ 2

Put them all in D.E.:

$$(x^2+1)\sum_{n=2}^{\infty} n(n-1) c_n x^{n-2} + x\sum_{n=1}^{\infty} n c_n x^{n-1} - \sum_{n=0}^{\infty} c_n x^n = 0$$

1

$$= \sum_{n=2}^{\infty} n(n-1) c_n x^n + \sum_{n=2}^{\infty} n(n-1) c_n x^{n-2} + \sum_{n=1}^{\infty} n c_n x^n - \sum_{n=0}^{\infty} c_n x^n$$

$$= 2c_2 x^0 - c_0 x^0 + 6c_3 x + c_1 x - c_1 x + \underbrace{\sum_{n=2}^{\infty} n(n-1) c_n x^n}_{k=n}$$

$$+ \underbrace{\sum_{n=4}^{\infty} n(n-1) c_n x^{n-2}}_{k=n-2} + \underbrace{\sum_{n=2}^{\infty} n c_n x^n}_{k=n} - \underbrace{\sum_{k=n} c_n x^n}_{k=n}$$ 3

$$= 2c_2 - c_0 + 6c_3 x + \sum_{k=2}^{\infty}\Big[k(k-1)c_k + (k+2)(k+1)c_{k+2} + kc_k - c_k\Big] x^k$$

$$= 2c_2 - c_0 + 6c_3 x + \sum_{k=2}^{\infty}\Big[(k+1)(k-1)c_k + (k+2)(k+1)c_{k+2}\Big] x^k = 0$$

Leads to: $\boxed{\begin{array}{l} 2c_2 - c_0 = 0 \\ 6c_3 = 0 \end{array}}$ 1 and $\boxed{(k+1)(k-1)c_k + (k+2)(k+1)c_{k+2} = 0}$ 1

$$\Rightarrow \quad c_2 = \frac{1}{2}c_0\,,\quad c_3 = 0\,,\quad \boxed{c_{k+2} = \frac{1-k}{k+2}\,c_k} \quad k=2,3,4,\dots$$

$$c_4 = -\frac{1}{4}c_2 = \frac{-1}{2.4}c_0 = \frac{-1}{2^2.2!}c_0$$

$$c_5 = \frac{-2}{5}c_3 = 0\,,\quad c_3 = 0 \Rightarrow \boxed{c_5 = 0}$$ 2

and odd terms are zero

$$c_6 = \frac{-3}{6}c_4 = \frac{3}{2.4.6}c_0 = \frac{1.3}{2^3\,3!} = c_0$$

$$c_7 = 0\,,\quad c_8 = \frac{-5}{8}c_6 = \frac{3.5}{2.4.6.8}c_0 = \frac{-1.3.5}{2^4\,4!}c_0$$

and so on, Now

$$y = c_0 + c_1x + c_2x^2 + c_3x^3 + c_4x^4 + c_5x^5 + \cdots$$

$$= c_1x + c_0\left[1 + \frac{1}{2}x^2 - \frac{1}{2^2 2!}x^4 + \frac{1.3}{2^3.3!}x^6 - \cdots\right]$$

1 1

$$= c_1y_1(x) + c_0y_2(x)$$

where $y_1(x) = x$

$$y_2(x) = 1 + \frac{1}{2}x^2 - \frac{1}{2^2 2!}x^4 + \cdots$$

6

2. (a) Determine the singular points of the differential equation

$$2x(x-2)^2 y'' + 3xy' + (x-2)y = 0$$

and classify them as regular or irregular.

Re-write: $y'' + \frac{3}{2(x-2)^2} y' + \frac{1}{2x(x-2)} y = 0$ 1

$P(x) = \frac{3}{2(x-2)^2}$ 1 and $q(x) = \frac{1}{2x(x-2)}$ 1

Singular Points: 0 and 2 1

$x=0$: $\lim_{x\to 0} xP(x) = 0 = \lim_{x\to 0} x^2 q(x)$: Exist 1

$x=2$: $\lim_{x\to 2} (x-2)P(x) = \lim_{x\to 2} \frac{3}{2(x-2)}$: D.N.E.

So, $x=0$ is Reg and $x=2$ is Irreg. 1

6

(b) $x = 0$ is a regular singular point of $x^2y'' + \left(\frac{5}{3}x + x^2\right) y' - \frac{1}{3}y = 0$. Use the general form of the indicial equation to find the indicial roots of the singularity. Without solving, discuss the number of a series solutions you would expect to find using the method of Frobenius.

Re-write $y'' + \left(\frac{\frac{5}{3}x + x^2}{x^2}\right) y' - \frac{1/3}{\frac{5}{3}x + x^2} y = 0$ 1

1 $P(x) = \frac{5}{3x+1}$ and 1 $Q(x) = \frac{-1}{3x^2}$ $\Rightarrow$

$\boxed{p(x) = \frac{5}{3} + x}$ and $\boxed{q(x) = -1/3}$ $\Rightarrow$ $\boxed{a_0 = 5/3}$ $\boxed{b_0 = -1/3}$ 1

Now, the Indicial eq.: 1

$r(r-1) + \frac{5}{3}r - 1/3 = r^2 + \frac{2}{3}r - \frac{1}{3} = \frac{1}{3}(3r-1)(r+1) = 0$

$\boxed{r_1 = 1/3}$ and $\boxed{r_2 = -1}$, they do not differ by an 1

Integer $\Rightarrow$ Expect 2-Series Solutions

3. Consider the system $X' = \begin{pmatrix} 0 & 6 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} X$

(a) Prove that the set of vectors

$$\left\{ \begin{pmatrix} 6 \\ -1 \\ -5 \end{pmatrix} e^{-t}, \begin{pmatrix} -3 \\ 1 \\ 1 \end{pmatrix} e^{-2t}, \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} e^{3t} \right\}$$

forms a Fundamental Set of Solutions.

I) Verify that X_1, X_2 and X_3 are Solutions

$$X_1' = \begin{pmatrix} -6 \\ +1 \\ +5 \end{pmatrix} e^{-t} = \begin{pmatrix} 0 & 6 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}\begin{pmatrix} 6 \\ -1 \\ -5 \end{pmatrix} e^{-t} \quad 2$$

$$X_2' = \begin{pmatrix} 6 \\ -2 \\ -2 \end{pmatrix} e^{-2t} = \begin{pmatrix} 0 & 6 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}\begin{pmatrix} -3 \\ 1 \\ 1 \end{pmatrix} e^{-2t} \quad 2$$

$$X_3' = \begin{pmatrix} 6 \\ 3 \\ 3 \end{pmatrix} e^{3t} = \begin{pmatrix} 0 & 6 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} e^{3t} \quad 2$$

II) Lin. Ind: show $W \neq 0$, for $t = 0$ 3

$$W = \begin{vmatrix} 6 & -3 & 2 \\ -1 & 1 & 1 \\ -5 & 1 & 1 \end{vmatrix} = 6\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} + 3\begin{vmatrix} -1 & 1 \\ -5 & 1 \end{vmatrix} + 2\begin{vmatrix} -1 & 1 \\ -5 & 1 \end{vmatrix}$$

$$= 0 + 3(-1+5) + 2(-1+5) = 20$$

(b) Form the general solution to the above system.

$$\boxed{X_g = c_1 X_1 + c_2 X_2 + c_3 X_3} \quad 3$$

4. Given the system

$$\frac{dx}{dt} = 6x - y$$

$$\frac{dy}{dt} = 5x + 2y$$

(a) Write the system in the Matrix form $X' = AX$.

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 6 & -1 \\ 5 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \quad 2$$

(b) Find the characteristic equation

$$\begin{vmatrix} 6-\lambda & -1 \\ 5 & 2-\lambda \end{vmatrix} = 0 = (6-\lambda)(2-\lambda) + 5 = 0$$

$$2 \qquad \boxed{\lambda^2 - 8\lambda + 17 = 0}$$

(c) Find the eigenvalues and there corresponding eigenvectors

Eigenv. : $\lambda^2 - 8\lambda + 17 = 0 \Rightarrow \lambda = \dfrac{8 \pm \sqrt{64 - 4(17)}}{2} = 4 \pm i$

$\lambda_1 = 4 + i$ 2 and $\lambda_2 = 4 - i$, Complex roots, conj.

$\lambda = 4 + i$: $(A - \lambda I)K_1 = 0 \Rightarrow \begin{pmatrix} 2-i & -1 \\ 5 & -2-i \end{pmatrix} \begin{pmatrix} k_1 \\ k_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$(2-i)k_1 - k_2 = 0$, $k_1 = 1 \Rightarrow k_2 = 2 - i$

$K_1 = \begin{pmatrix} 1 \\ 2-i \end{pmatrix}$, then 2 $K_2 = \begin{pmatrix} 1 \\ 2+i \end{pmatrix}$

(d) Find the general solution to the system.

$$X_g = C_1 X_1 + C_2 X_2 = C_1 \begin{pmatrix} 1 \\ 2-i \end{pmatrix} e^{(4+i)t} + C_2 \begin{pmatrix} 1 \\ 2+i \end{pmatrix} e^{(4-i)t}$$

$$= C_1 \begin{pmatrix} \sin t - \cos t \\ 2\cos t \end{pmatrix} e^{4t} + C_2 \begin{pmatrix} -\sin t - \cos t \\ 2\sin t \end{pmatrix} e^{4t}$$

2 2

5. Solve the system $X' = \begin{pmatrix} -1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 3 & -1 \end{pmatrix} X$, knowing that $\lambda = -1$ is an eigenvalue of A.

$A = \begin{pmatrix} -1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 3 & -1 \end{pmatrix}$, cha. eq: $|A - \lambda I| = 0$

Exp. around R_1: $\begin{vmatrix} -1-\lambda & 1 & 0 \\ 1 & 2-\lambda & 1 \\ 0 & 3 & -1-\lambda \end{vmatrix} = 0$ 1

$(-1-\lambda)\begin{vmatrix} 2-\lambda & 1 \\ 3 & -1-\lambda \end{vmatrix} - \begin{vmatrix} 1 & 1 \\ 6 & -1-\lambda \end{vmatrix} = 0 = (1+\lambda)(\lambda^2 - \lambda - 6) = 0$

$\lambda_1 = -1$, $\lambda_2 = 3$ and $\lambda_3 = -2$ 1 1

$\lambda_1 = -1 \Rightarrow \begin{pmatrix} 0 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 3 & 0 \end{pmatrix}\begin{pmatrix} K_1 \end{pmatrix} = 0$, $K_1 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ 2

$\lambda_2 = 3 \Rightarrow (A - 3I)K_2 = 0 \Longrightarrow K_2 = \begin{pmatrix} 1 \\ 4 \\ 3 \end{pmatrix}$ 2

$\lambda_3 = -2 \Rightarrow (A + 2I)K_3 = 0 \Rightarrow K_3 = \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$ 2

$X_g = c_1 K_1 e^{-t} + c_2 K_2 e^{3t} + c_3 K_3 e^{-2t}$ 3

6. Solve the Initial-Value-Problem

$$X' = \begin{pmatrix} 2 & 4 \\ -1 & 6 \end{pmatrix} X, \quad X(0) = \begin{pmatrix} -1 \\ 6 \end{pmatrix}$$

$A = \begin{pmatrix} 2 & 4 \\ -1 & 6 \end{pmatrix}$, $\begin{vmatrix} 2-\lambda & 4 \\ -1 & 6-\lambda \end{vmatrix} = 0 = (2-\lambda)(6-\lambda)+4$ 1

$= (\lambda-4)^2$, Repeated roots $\lambda_1 = \lambda_2 = 4$ 2

$\lambda_1 = 4 : (A-4I)K_1 = 0 \Longrightarrow K_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ 1

Now, $(A-4I)K_2 = K_1$ $\begin{pmatrix} -2 & 4 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} k_1 \\ k_2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$-2k_1 + 4k_2 = 2 \Longrightarrow -k_1 + 2k_2 = 1$, if $k_2 = 1 \Rightarrow k_1 = 1$

So $K_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ 1

$X_g = c_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{4t} + c_2 \left[\begin{pmatrix} 2 \\ 1 \end{pmatrix} t e^{4t} + \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{4t} \right]$

1 3

Now, $X(0) = \begin{pmatrix} -1 \\ 6 \end{pmatrix}$

1 $\begin{pmatrix} -1 \\ 6 \end{pmatrix} = \begin{pmatrix} 2c_1 \\ c_1 \end{pmatrix} + \begin{pmatrix} c_2 \\ c_2 \end{pmatrix} \Rightarrow \begin{matrix} -1 = 2c_1 + c_2 \\ 6 = c_1 + c_2 \end{matrix}$

$-1 - 2c_1 = 6 - c_1 \Longrightarrow \boxed{c_1 = -7}$ and

$\boxed{c_2 = 13}$

1 1

7. Use the variation of parameters to solve the system

$$X' = \begin{pmatrix} 1 & 8 \\ 1 & -1 \end{pmatrix} X + \begin{pmatrix} 12 \\ 12 \end{pmatrix} t$$

$A = \begin{bmatrix} 1 & 8 \\ 1 & -1 \end{bmatrix} \Rightarrow |A - \lambda I| = 0 = \begin{vmatrix} 1-\lambda & 8 \\ 1 & -1-\lambda \end{vmatrix} = 0 = \lambda^2 - 9 = 0$

$\lambda_1 = 3$ and $\lambda_2 = -3$ 2 $\lambda = \pm 3$

$\Downarrow$

$K_1 = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$ and $K_2 = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$ 2 $\begin{vmatrix} 4 & -2 \\ 1 & 1 \end{vmatrix} = 6$

$X_c = c_1 \begin{pmatrix} 4 \\ 1 \end{pmatrix} e^{3t} + c_2 \begin{pmatrix} -2 \\ 1 \end{pmatrix} e^{-3t}$

Now $\phi(t) = \begin{pmatrix} 4e^{3t} & -2e^{-3t} \\ e^{3t} & e^{-3t} \end{pmatrix} \Rightarrow$ 2

$\phi^{-1}(t) = \begin{pmatrix} 1/6\, e^{-3t} & 1/3\, e^{-3t} \\ -1/6\, e^{3t} & 2/3\, e^{3t} \end{pmatrix}$ 1

Now $U = \int \phi^{-1} \cdot F\, dt = \int \begin{pmatrix} 6t e^{-3t} \\ 6t e^{3t} \end{pmatrix} dt$ 1

by Parts $= \begin{pmatrix} -2t e^{-3t} - \frac{2}{3} e^{-3t} \\ 2t e^{3t} - 2/3\, e^{3t} \end{pmatrix}$ 2

Now, $X_p = \phi U = \begin{pmatrix} -12 \\ 0 \end{pmatrix} t + \begin{pmatrix} -4/3 \\ -4/3 \end{pmatrix}$ 2

$\boxed{X_g = X_c + X_p}$

8. Given the Matrix $A = \begin{pmatrix} 0 & 0 & 0 \\ 3 & 0 & 0 \\ 5 & 1 & 0 \end{pmatrix}$

a) Compute A^2

2

$$= \begin{pmatrix} 0 & 0 & 0 \\ 3 & 0 & 0 \\ 5 & 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 & 0 \\ 3 & 0 & 0 \\ 5 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 3 & 0 & 0 \end{pmatrix}$$

(b) Compute A^3

2

$$= A^2 A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 3 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 & 0 \\ 3 & 0 & 0 \\ 5 & 1 & 0 \end{pmatrix} = 0_{3\times 3}$$

and so, $A^4 = A^5 = \cdots = 0_{3\times 3}$

(c) Find e^{At}

$$e^{At} = I + At + \frac{1}{2}A^2t^2 =$$ 3

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 3t & 0 & 0 \\ 5t & t & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{3}{2}t^2 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 3t & 1 & 0 \\ \frac{3}{2}t^2+5t & t & 1 \end{pmatrix}$$

(d) Find $\frac{d}{dt}(e^{At})$

$$= Ae^{At}$$ 2

(e) Use parts (c) and (d) to find the general solutions of $X' = AX$.

$$X = \begin{pmatrix} 1 & 0 & 0 \\ 3t & 1 & 0 \\ \frac{3}{2}t^2+5t & t & 1 \end{pmatrix}\begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = c_1\begin{pmatrix} 1 \\ 3t \\ \frac{3}{2}t^2+5t \end{pmatrix} + c_2\begin{pmatrix} 0 \\ 1 \\ t \end{pmatrix} + c_3\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

3

9. Solve the following first order differential equations:

4 (a) $\frac{dy}{dx} = \frac{xy + 3x - y - 3}{xy - 2x + 4y - 8} = \frac{x(y+3) - (y+3)}{x(y-2) + 4(y-2)} \Leftrightarrow$

$\frac{dy}{dx} = \frac{(y+3)(x-1)}{(y-2)(x+4)}$ 1 Sep. D.E.

$\left(\frac{y-2}{y+3}\right) dy \overset{1}{=} \left(\frac{x-1}{x+4}\right) dx \Leftrightarrow \left(1 + \frac{2}{y-1}\right) dy = \left(1 + \frac{5}{x-3}\right) dx$

$y + 2\ln|y-1| = x + 5\ln|x-3| + c$ 1 1

OR $\frac{(y-1)^2}{(x-3)^5} = c\, e^{x-y}$

4 (b) $\cos x \frac{dy}{dx} + (\sin x)y = 1$

$\frac{dy}{dx} + (\tan x)y = \sec x$ 1 L. D. E.

$\mu(x) = e^{\int \tan x\, dx} = \sec x$ 1 $\Leftarrow$ $\boxed{\int \frac{\sin x}{\cos x} dx}$

$\frac{d}{dx}(y \cdot \sec x) = \sec^2 x$ 1 $\Leftarrow$ $\int \sec^2 x = \tan x$

$\boxed{y = \sin x + C \cdot \cos x}$ 1 For simp, consider $|x| < \pi/2$

4 (c) $\frac{dy}{dx} = y(xy^3 - 1) \Leftrightarrow y' + y = xy^4$, Ber. with $n = 4$ 1

let $w = y^{-3}$ 1 $\Longrightarrow \frac{dw}{dx} - 3w = -3x$ (Lin) 1

$\mu(x) = e^{-3x} \Longrightarrow e^{-3x} w = xe^{-3x} + \frac{1}{3} e^{-3x} + c$

$\Leftarrow$ $\boxed{y^{-3} = x + \frac{1}{3} + ce^{3x}}$ 1

10. Solve the Initial Value Problem

$$y''' + 2y'' - 5y' - 6y = 0, \quad y(0) = y'(0) = y''(0) = 1.$$

Aux. Eq. 1 $m^3 + 2m^2 - 5m - 6 = 0$, Try ± 1

$m = -1$ is one root 1

1

$$\begin{array}{r} m^2 + m - 6 \\ m+1 \overline{)\, m^3 + 2m^2 - 5m - 6} \\ m^3 + m^2 \\ \hline m^2 - 5m - 6 \\ m^2 + m \\ \hline -6m - 6 \\ -6m - 6 \\ \hline 0 \end{array}$$

$m^2 + m - 6 = (m+3)(m-2)$ 2

$m = -1, -3, 2$

$\boxed{y = c_1 e^{-3t} + c_2 e^{-t} + c_3 e^{2t}}$ 2

$y' = -3c_1 e^{-3t} - c_2 e^{-t} + 2c_3 e^{2t}$ 1

$y'' = 9c_1 e^{-3t} + c_2 e^{-t} + 4c_3 e^{2t}$ 1

Let $t = 0$ in y, y' and y'' Leads to:

$$\left.\begin{array}{l} c_1 + c_2 + c_3 = 1 \quad (1) \\ -3c_1 - c_2 + 2c_3 = 1 \quad (2) \\ 9c_1 + c_2 + 4c_3 = 1 \quad (3) \end{array}\right\}$$

(1)+(2) and (2)+(3) $\Rightarrow$

$-2c_1 + 3c_3 = 2$ and $6c_1 + 6c_3 = 2$

Solve $\Rightarrow$ $\boxed{c_3 = 8/15}$ 1

$\Rightarrow$ $\boxed{c_1 = -1/5}$ 1

In (1) $-\frac{1}{5} + c_2 + \frac{8}{15} = 1 \Rightarrow \boxed{c_2 = 2/3}$

1

11. The complementary solution of

$$y''' - y'' + y' - y = xe^{x} - e^{-x} + 7$$

is $y_c = c_1e^x + c_2 \cos x + c_3 \sin x$. Determine the form of the particular solution y_p, without evaluating the constants.

The Annihilator of $xe^{x} - e^{-x} + 7$ is
(1) (1) (1)
$D(D-1)^2(D+1)$

(1) (1) (1)

So $D(D-1)^2(D+1) = 0 \implies D = 0, 1, 1, -1$

$y_p = c_4 + c_5 e^{-x} + c_6 xe^{x} + c_7 x^2 e^{x}$
(1) (1) (1) (1)

Notice that e^{x} is in the comp. Solution

12. Solve by variation of parameters $y'' - y = \cosh x$, where $\cosh x = \dfrac{e^{x}+e^{-x}}{2}$.

Aux. Eq: $m^2 - 1 = 0 \Rightarrow m = \pm 1 \Rightarrow$ 1

2 $y_c = c_1 e^{x} + c_2 e^{-x}$, $W(e^{x}, e^{-x}) = \begin{vmatrix} e^{x} & e^{-x} \\ e^{x} & -e^{-x} \end{vmatrix} = -2$ 1

and $f(x) = \frac{1}{2}(e^{x} + e^{-x})$

$W_1 = \begin{vmatrix} 0 & e^{-x} \\ \cosh x & -e^{-x} \end{vmatrix} = +\frac{1}{2}(1 + e^{-2x}) \Rightarrow u_1' = \frac{1}{4} e^{-2x} + \frac{1}{4}$ 1

$W_2 = \begin{vmatrix} e^{x} & 0 \\ e^{x} & \cosh x \end{vmatrix} = -\frac{1}{2}(e^{2x} + 1) \Rightarrow u_2' = -\frac{1}{4} e^{2x} - \frac{1}{4}$ 1

$\Rightarrow$ 1 $\boxed{u_1 = -\frac{1}{8} e^{-2x} + \frac{1}{4} x}$ and 1 $\boxed{u_2 = -\frac{1}{8} e^{2x} - \frac{1}{4} x}$

$y_p = u_1 y_1 + u_2 y_2$ where $y_1 = e^{-x}$ and $y_2 = e^{+x}$

$= \left(-\frac{1}{8} e^{-2x} + \frac{1}{4} x\right) e^{-x} + \left(-\frac{1}{8} e^{2x} - \frac{1}{4} x\right) e^{+x}$

$y_g = y_c + y_p$ 2