Math201.05, Quiz #2, Term 161

Name: Solutions

ID #: **Serial #:**

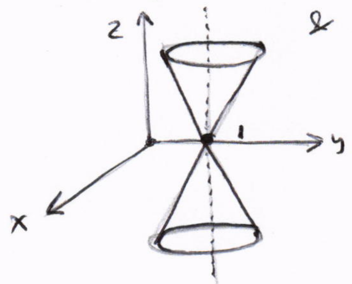
1. **[2.5 points]** Identify and sketch the graph of the equation $2x^2 + y^2 - 2y - z^2 = -1$.
2. **[2.5 points]** Find and sketch the domain of $f(x, y) = \sqrt{y + x} + 2\ln(y - x)$.
3. **[2.5 points]** Find the limit: $\lim_{(x,y)\to(0,0)} \frac{1-\cos(x^2+y^2)}{sin^2(2x^2+2y^2)}$.
4. **[2.5 points]** Let $z = tan^{-1}\left(\frac{x+y}{1-xy}\right)$. Find $\frac{\partial^2 z}{\partial y \partial x}$. Simplify your answer.

Good luck,

Ibrahim Al-Rasasi

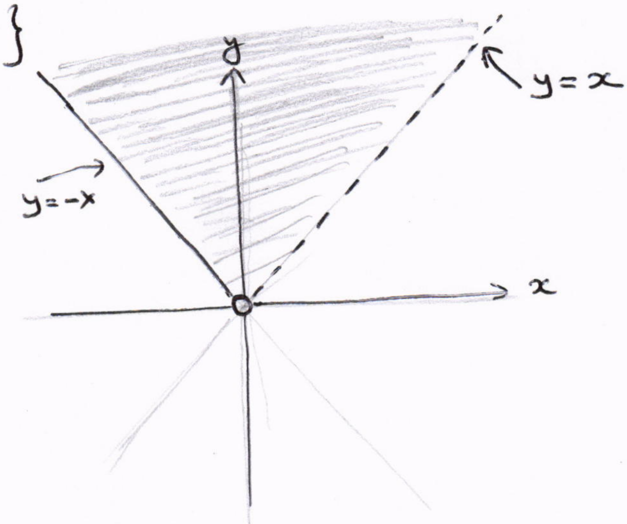
$\boxed{1}$ $2x^2+y^2-2y-z^2=-1 \Rightarrow 2x^2+y^2-2y+1=z^2 \Rightarrow 2x^2+(y-1)^2=z^2$

an elliptic cone with vertex $(0,1,0)$, axis: the line through $(0,1,0)$ & parallel to the z-axis.



$\boxed{2}$ Domain $= \{(x,y) : y+x \geq 0$ and $y-x>0\}$

$= \{(x,y) : y \geq -x$ and $y > x\}$



$\boxed{3}$ Use polar Coordinates:

$$\lim_{(x,y)\to(0,0)} \frac{1-\cos(x^2+y^2)}{\sin^2(2x^2+2y^2)} = \lim_{\substack{r\to 0^+\\ \theta}} \frac{1-\cos(r^2)}{\sin^2(2r^2)} \quad , \ \frac{0}{0} \Longrightarrow \text{L'Hospital's Rule}$$

$$= \lim_{r\to 0^+} \frac{\sin(r^2)\cdot 2r}{2\sin(2r^2)\cdot\cos(2r^2)\cdot 4r}$$

$$\underline{\text{Simplify}} \lim_{r\to 0^+} \frac{1}{4\cos(2r^2)}$$

$$= \frac{1}{4\cdot 1} = \frac{1}{4}$$

$\boxed{4}$ $z = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$

- $$\frac{\partial z}{\partial x} = \frac{1}{1+\left(\frac{x+y}{1-xy}\right)^2} \cdot \frac{(1-xy)(1)-(x+y)(-y)}{(1-xy)^2}$$

$$= \frac{1-xy+xy+y^2}{(1-xy)^2+(x+y)^2}$$

$$= \frac{1+y^2}{1-2xy+x^2y^2+x^2+2xy+y^2} = \frac{1+y^2}{1+x^2y^2+x^2+y^2}$$

$$= \frac{1+y^2}{(1+y^2)+x^2(1+y^2)} = \frac{1+y^2}{(1+y^2)(1+x^2)}$$

$$= \frac{1}{1+x^2}$$

- $$\frac{\partial z}{\partial y\,\partial x} = 0$$

Math201.10, Quiz #2, Term 161

Name:

ID #: **Serial #:**

1. **[2.5 points]** Identify and sketch the graph of the equation $z^2 = 2x^2 + y^2 - 2y + 2$.
2. **[2.5 points]** Find and sketch the domain of $f(x,y) = \frac{\sqrt{y-x^2}}{y-x}$.
3. **[2.5 points]** Find the limit: $\lim_{(x,y)\to(0,0)} \frac{xy^2e^x}{y^4+2x^2}$.
4. **[2.5 points]** Let $z = tan^{-1}\left(\frac{x+y}{1-xy}\right)$. Find $\frac{\partial^2 z}{\partial x^2}$. Simplify your answer.

Good luck,

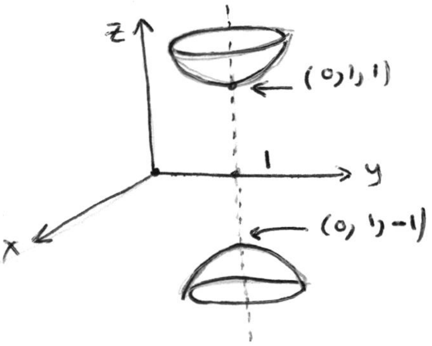
Ibrahim Al-Rasasi

$\boxed{1}$ $z^2 = 2x^2 + y^2 - 2y + 2 \Rightarrow z^2 = 2x^2 + y^2 - 2y + 1 + 1 \Rightarrow z^2 = 2x^2 + (y-1)^2 + 1$

$\Rightarrow -2x^2 - (y-1)^2 + z^2 = 1$

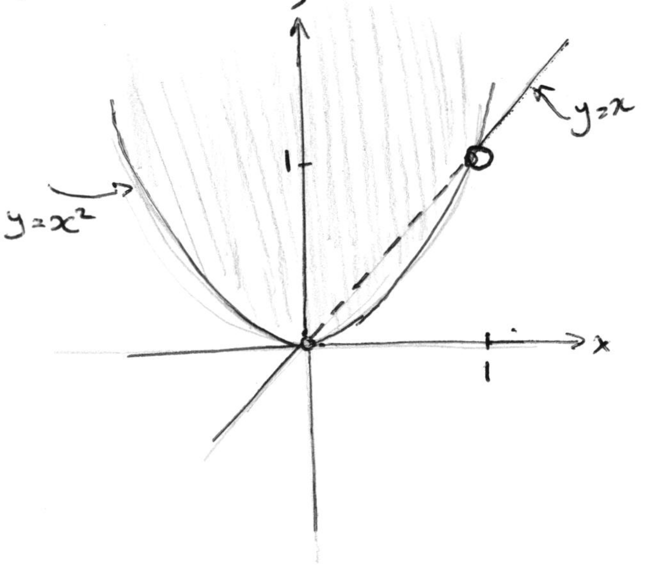
a hyperboloid of two sheets

axis: the line through $(0,1,0)$ & parallel to the z-axis.



$\boxed{2}$ Domain $= \{(x,y) : y - x^2 \geq 0 \text{ and } y - x \neq 0\}$

$= \{(x,y) : y \geq x^2 \text{ and } y \neq x\}$



[3] $\lim\limits_{(x,y)\to(0,0)} \dfrac{xy^2 e^x}{y^4+2x^2}$

- along the x-axis : $y=0$

$$\lim_{\substack{(x,y)\to(0,0)\\ y=0}} \frac{xy^2 e^x}{y^4+2x^2} = \lim_{(x,y)\to(0,0)} \left.\frac{xy^2 e^x}{y^4+2x^2}\right|_{y=0} = \lim_{(x,y)\to(0,0)} \frac{0}{2x^2} = \lim_{(x,y)\to(0,0)} 0 = 0$$

- along the parabola $x=y^2$

$$\lim_{\substack{(x,y)\to(0,0)\\ x=y^2}} \frac{xy^2 e^x}{y^4+2x^2} = \lim_{(x,y)\to(0,0)} \left.\frac{xy^2 e^x}{y^4+2x^2}\right|_{x=y^2} = \lim_{y\to 0} \frac{y^4 e^{y^2}}{3y^4} = \lim_{y\to 0} \frac{1}{3} e^{y^2} = \frac{1}{3}\cdot 1 = \frac{1}{3}$$

Since the limits along the two paths are not equal, then the given limit does not exists.

[4] $Z = \tan^{-1}\left(\dfrac{x+y}{1-xy}\right)$

- $\dfrac{\partial Z}{\partial x} = \dfrac{1}{1+x^2}$ (see the Solution in page 2)

- $\dfrac{\partial^2 Z}{\partial x^2} = \dfrac{(1+x^2)\cdot(0) - (1)(2x)}{(1+x^2)^2}$

$$= \frac{-2x}{(1+x^2)^2}$$