HW

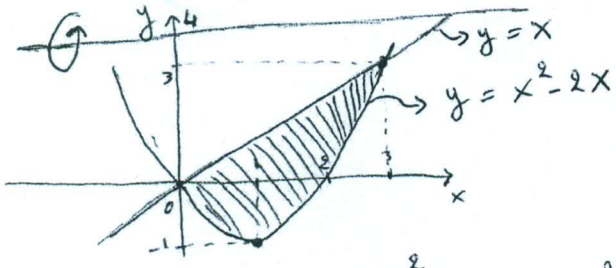
Exercise 1: (5 points)
Determine the volume of the solid obtained by rotating the region bounded by $y = x$ and $y = x^2 - 2x$ about the line y = 4

Exercise 2: (5 points)
Determine the volume of the solid obtained by rotating the region bounded by $y = x - 1$ and $y = 2\sqrt{x-1}$ about the line x = -1

Exercise 3: (5 points)
Determine the volume of the solid obtained by rotating the region bounded by $x = y^2$ and $x = y$ about the line y = 1

Exercise 4: (5 points)
The base of a solid is a semi-circle of radius 1. Parallel cross-sections perpendicular to the base are squares with two of their vertices on the semi-circle. Find the volume of the solid.

exercise 1

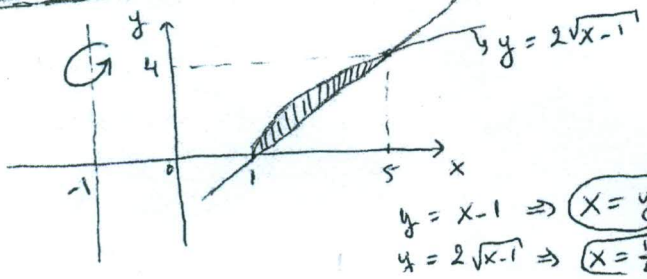


$$V = \int_0^3 \pi \left(4 - (x^2 - 2x)\right)^2 - \pi (4 - x)^2 \, dx$$

$$= \int_0^3 \pi \left(x^4 - 4x^3 - 5x^2 + 24x\right) dx$$

$$= \pi \left[\frac{x^5}{5} - x^4 - \frac{5x^3}{3} + 12x^2\right]_0^3 = \frac{153}{5}\pi$$

exercise 2



$y = x - 1 \Rightarrow x = y + 1$

$y = 2\sqrt{x-1} \Rightarrow x = \frac{1}{4}y^2 + 1$

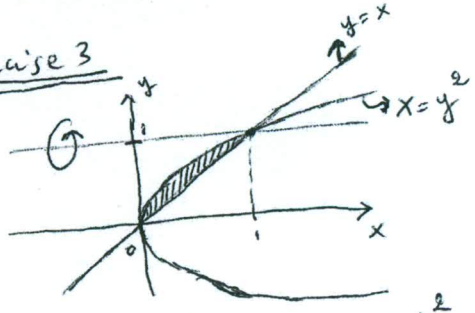
$$V = \int_0^4 \pi \left(y + 1 - (-1)\right)^2 - \pi \left(\frac{1}{4}y^2 + 1 - (-1)\right)^2 dy$$

$$= \int_0^4 \pi (y+2)^2 - \pi \left(\frac{1}{4}y^2 + 2\right)^2 dy$$

$$= \pi \int_0^4 4y - \frac{y^4}{16} \, dy$$

$$= \pi \left[2y^2 - \frac{1}{80}y^5\right]_0^4 = \frac{96}{5}\pi$$

exercise 3



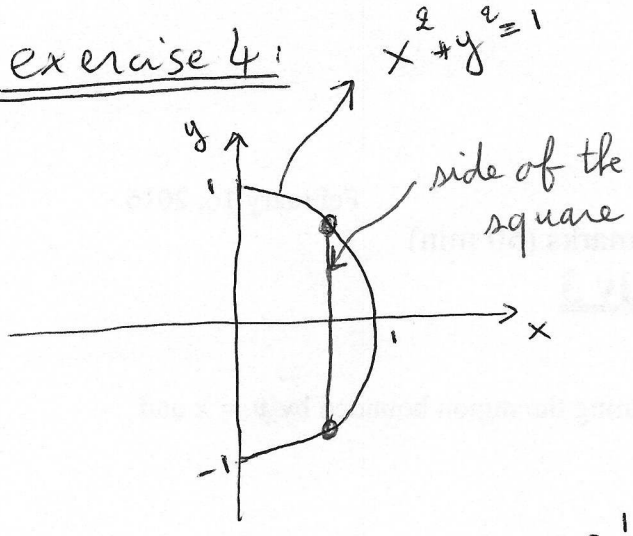
$$V = \int_0^1 \pi (1 - x)^2 - \pi \left(1 - \sqrt{x}\right)^2 dx$$

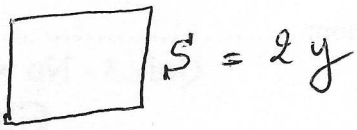
$$= \pi \int_0^1 1 - 2x + x^2 - 1 + 2\sqrt{x} - x \, dx$$

$$= \pi \int_0^1 x^2 - 3x + 2\sqrt{x} \, dx$$

$$= \pi \left[\frac{x^3}{3} - \frac{3}{2}x^2 + \frac{4}{3}x^{3/2}\right]_0^1 = \frac{\pi}{6}$$

exercise 4:





$$V = \int_0^1 \underbrace{A(x)}_{\text{area of the square}} dx = \int_0^1 S^2 \, dx = \int_0^1 (2y)^2 \, dx$$

$$= \int_0^1 4y^2 \, dx = \int_0^1 4(1-x^2) \, dx = \left[4x - \frac{4}{3}x^3\right]_0^1$$

$$= \boxed{\frac{8}{3}}$$