MATH 102.1 (Term 161)

Quiz 5 (Sects. 11.1, 11.2 & 11.3) Duration: 20min

Name: ID number:

1.)(2pts)Find the limit of the sequence $\left\{n\left(\sqrt{n^2+1}-n\right)\right\}_{n=1}^{\infty}$.

2.)(4pts)Evaluate the sums $\sum_{n=1}^{\infty}\frac{2^{2n-1}}{7^{n+1}}$, $\sum_{n=3}^{\infty}\frac{1}{n^2-n-2}$.

3.)(4pts)Do the series converge or diverge $\sum_{n=1}^{\infty}\frac{e^n}{e^{2n}+1}$, $\sum_{n=2}^{\infty}\left(\frac{1}{n^2}+\frac{n^3}{n^4+1}\right)$?

1.) $n(\sqrt{n^2+1}-n) = n\,\frac{n^2+1-n^2}{\sqrt{n^2+1}+n}$

$= \frac{1}{\sqrt{1+\frac{1}{n^2}}+1}$

$\lim_{n\to\infty} n(\sqrt{n^2+1}-n) = \frac{1}{2}$

2.) $\sum_{n=1}^{\infty}\frac{2^{2n-1}}{7^{n+1}} = \sum_{n=1}^{\infty}\frac{1}{14}\left(\frac{4}{7}\right)^n = \sum_{n=1}^{\infty}\frac{1}{14}\frac{4}{7}\left(\frac{4}{7}\right)^{n-1}$

$= \frac{\frac{1}{14}\cdot\frac{4}{7}}{1-\frac{4}{7}} = \frac{\frac{2}{7.7}}{3/7}$

$= \frac{2}{21}$

$a_n = \frac{3}{n^2-n-2} = \frac{3}{(n+1)(n-2)} = \frac{1}{n-2} - \frac{1}{n+1}$

$S_n = a_3 + a_4 + a_5 + \cdots + a_n$

$= \left(1-\cancel{\frac{1}{4}}\right) + \left(\frac{1}{2}-\cancel{\frac{1}{5}}\right) + \left(\frac{1}{3}-\cancel{\frac{1}{6}}\right) + \left(\cancel{\frac{1}{4}}-\cancel{\frac{1}{7}}\right) + \cdots +$

$+\left(\cancel{\frac{1}{n-5}}-\cancel{\frac{1}{n-2}}\right) + \left(\cancel{\frac{1}{n-4}}-\frac{1}{n-1}\right) + \left(\cancel{\frac{1}{n-3}}-\frac{1}{n}\right) + \left(\cancel{\frac{1}{n-2}}-\frac{1}{n+1}\right)$

$= 1+\frac{1}{2}+\frac{1}{3}-\frac{1}{n-1}-\frac{1}{n}-\frac{1}{n+1}$

$\lim_{n\to\infty} S_n = \frac{11}{6} \Rightarrow \boxed{\sum_{n=3}^{\infty}\frac{1}{n^2-n-n} = \frac{11}{18}}$

3.) $f(x) = \frac{e^x}{e^{2x}+1}$ is positive

$f'(x) = \frac{e^x(1-e^{2x})}{(e^{2x}+1)^2} < 0,\ x>0$

From integral test, the serie $\sum\frac{e^n}{e^{2n}+1}$ and the integral $\int_1^{\infty} f(x)\,dx$ have the same nature.

Now, $\int_1^{\infty}\frac{e^x}{e^{2x}+1}\,dx = \lim_{b\to\infty}\left[\tan^{-1}(e^x)\right]_1^b$

$= \pi/2 - \tan^{-1}(e)$

$\Rightarrow \sum_{n=1}^{\infty}\frac{e^n}{e^{2n}+1}$ converges

b) $f(x) = \frac{x^3}{x^4+1}$, $f' = \frac{x^2(3-x^4)}{(x^4+1)^2} < 0,\ x>2$

$\int_1^{\infty}\frac{x^3}{x^4+1}\,dx = \lim_{b\to\infty}\frac{1}{4}\left[\ln(x^4+1)\right]_1^b = \infty$

$\sum\frac{n^3}{n^4+1}$ diverges by integral test

$\sum\frac{1}{n^2}$ converges

$\Rightarrow \sum_{n=1}^{\infty}\left(\frac{1}{n^2}+\frac{n^3}{n^4+1}\right)$ diverges

MATH 102.3 (Term 161)

Quiz 5 (Sects. 11.1, 11.2 & 11.3) **Duration: 20min**

Name: **ID number:**

1.)(2pts)Find the limit of the sequence $\left\{n^{3/2}\left(\frac{1}{\sqrt{n+2}}-\frac{1}{\sqrt{n}}\right)\right\}_{n=1}^{\infty}$.

2.)(4pts)Evaluate the sums $\sum_{n=1}^{\infty}\frac{2^{3n-1}}{3^{2n+1}}$, $\sum_{n=2}^{\infty}\frac{1}{n^2+n-2}$.

3.)(4pts)Do the series converge or diverge $\sum_{n=1}^{\infty}\frac{1}{e^{-n}+e^{n}}$, $\sum_{n=2}^{\infty}\frac{n^2+2n}{n^2+2n+1}$?

1.) $n^{3/2}\left(\frac{1}{\sqrt{n+2}}-\frac{1}{\sqrt{n}}\right)=\frac{n^{3/2}}{\sqrt{n}\sqrt{n+2}}\left(\sqrt{n}-\sqrt{n+2}\right)$

$=\frac{n}{\sqrt{n+2}}\;\frac{n-(n+2)}{\sqrt{n}+\sqrt{n+2}}$

$=\frac{-2}{\sqrt{1+\frac{2}{n}}\;\left(1+\sqrt{1+\frac{2}{n}}\right)}$

$\lim_{n\to\infty} n^{3/2}\left(\frac{1}{\sqrt{n+2}}-\frac{1}{\sqrt{n}}\right)=-1$

2.) $\sum_{n=1}^{\infty}\frac{2^{3n-1}}{3^{2n+1}}=\sum_{n=1}^{\infty}\frac{1}{6}\left(\frac{8}{9}\right)^n$

$=\sum_{n=1}^{\infty}\frac{1}{6}\left(\frac{8}{9}\right)\left(\frac{8}{9}\right)^{n-1}$

$=\frac{4/27}{1-8/9}=\frac{4}{3}$

$a_n=\frac{3}{n^2+n-2}=\frac{3}{(n-1)(n+2)}=\frac{1}{n-1}-\frac{1}{n+2}$

$S_n=a_2+a_3+a_4+\dots+a_n$

$=\left(1-\cancel{\frac{1}{4}}\right)+\left(\frac{1}{2}-\cancel{\frac{1}{5}}\right)+\left(\frac{1}{3}-\cancel{\frac{1}{6}}\right)+\left(\cancel{\frac{1}{4}}-\cancel{\frac{1}{7}}\right)+\dots$

$+\left(\cancel{\frac{1}{n-4}}-\cancel{\frac{1}{n-1}}\right)+\left(\cancel{\frac{1}{n-3}}-\frac{1}{n}\right)+\left(\cancel{\frac{1}{n-2}}-\frac{1}{n+1}\right)+\left(\cancel{\frac{1}{n-1}}-\frac{1}{n+2}\right)$

$=1+\frac{1}{2}+\frac{1}{3}-\frac{1}{n}-\frac{1}{n+1}-\frac{1}{n+2}$

$\Rightarrow \lim_{n\to\infty} S_n=$ [illegible] $\Rightarrow \boxed{\sum_{n=2}^{\infty}\frac{1}{n^2+n-2}=\frac{[illegible]}{18}}$

3.) $f(x)=\frac{1}{e^{-x}+e^{x}}=\frac{e^x}{e^{2x}+1}$ is positive

a) $f'(x)=\frac{e^x(1-e^{2x})}{(e^{2x}+1)^2}<0,\; x>0$

$\int_1^{\infty}\frac{e^x}{e^{2x}+1}\,dx=\lim_{b\to\infty}\left[\tan^{-1}(e^x)\right]_1^b$

$=\pi/2-\tan^{-1}(e)$

$\Rightarrow \sum_{n=1}^{\infty}\frac{1}{e^{-n}+e^{n}}$ converges by the integral test

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b) $a_n=\frac{n^2+2n}{n^2+2n+1}$

$\lim_{n\to\infty} a_n=1\neq 0$

$\Rightarrow \sum_{n=1}^{\infty}\frac{n^2+2n}{n^2+2n+1}$ diverges by the n^{th} term test for divergence.