MATH 102.1 (Term 161)

Quiz 4 (Sects. 7.4, 7.8 & 8.1) Duration: 20min

Name: ID number:

---

1.)( pts)Evaluate the integral $I = \int \frac{1}{x(x^2-2x-3)}dx$.

2.)( pts)Does the integral $J = \int_0^1 \frac{\ln x}{x}dx$ converge or diverge?

3.)(3pts)Find the arc length of the curve $y = \frac{x^4}{4} + \frac{1}{8x^2}$ for $1 \le x \le 2$.

---

1.) $\frac{1}{x(x^2-2x-3)} = \frac{a}{x} + \frac{b}{x+1} + \frac{c}{x-3}$

$a = \frac{1}{x^2-2x-3}\Big|_{x=0} = -\frac{1}{3}$

$b = \frac{1}{x(x-3)}\Big|_{x=-1} = 1/4$

$c = \frac{1}{x(x+1)}\Big|_{x=3} = \frac{1}{12}$

$I = \int \left(\frac{-\frac{1}{3}}{x} + \frac{\frac{1}{4}}{x+1} + \frac{\frac{1}{12}}{x-3}\right)dx$

$= -\frac{1}{3}\ln|x| + \frac{1}{4}\ln|x+1| + \frac{1}{12}\ln|x-3| + C$

2.) $\int_t^1 \frac{\ln x}{x}dx = \left[\frac{(\ln x)^2}{2}\right]_t^1$

$= -\frac{(\ln t)^2}{2}$

$\lim_{t\to 0^+}\left(-\frac{(\ln t)^2}{2}\right) = -\infty$

Thus, J diverges.

3.) $f(x) = \frac{x^4}{4} + \frac{1}{8x^2}$

$f'(x) = x^3 - \frac{1}{4x^3}$

$1 + f'(x)^2 = 1 + \left(x^3 - \frac{1}{4x^3}\right)^2$

$= 1 + \left(x^6 - \frac{1}{2} + \frac{1}{16x^6}\right)$

$= x^6 + \frac{1}{2} + \frac{1}{16x^6}$

$= \left(x^3 + \frac{1}{4x^3}\right)^2$

$L = \int_1^2 \left(x^3 + \frac{1}{4x^3}\right)dx$

$= \left[\frac{x^4}{4} - \frac{1}{8}\frac{1}{x^2}\right]_1^2$

$= 4 - \frac{1}{32} - \frac{1}{4} + \frac{1}{8}$

$= \frac{123}{32}$.

MATH 102.3 (Term 161)

Quiz 4 (Sects. 7.4, 7.8 & 8.1) Duration: 20min

Name: ID number:

1.)(4pts)Evaluate the integral $I = \int \frac{1}{x(2x^2+x+1)}dx$.

2.)(3pts)Does the integral $J = \int_2^3 \frac{1}{(x-2)^{3/2}}dx$ converge or diverge?

3.)(3pts)Find the arc length of the curve $y = \frac{1}{2}(\frac{3}{2}e^{2x/3} + \frac{3}{2}e^{-2x/3})$ for $0 \leq x \leq 1$.

---

1.) $\frac{1}{x(2x^2+x+1)} = \frac{a}{x} + \frac{bx+c}{2x^2+x+1}$

$= \frac{(2a+b)x^2+(a+c)x+a}{x(2x^2+x+1)}$

$\begin{cases} 2a+b=0 \to b=-2 \\ a+c=0 \to c=-1 \\ a=1 \end{cases}$

$I = \int\left(\frac{1}{x} - \frac{2x+1}{2x^2+x+1}\right)dx$

$= \int\left(\frac{1}{x} - \frac{1}{2}\frac{4x+2}{2x^2+x+1}\right)dx$

$= \int\left(\frac{1}{x} - \frac{1}{2}\frac{4x+1}{2x^2+x+1} - \frac{1}{4}\frac{1}{(x+\frac{1}{4})^2+\frac{7}{16}}\right)dx$

$= \ln|x| - \frac{1}{2}\ln(2x^2+x+1) - \frac{4}{4\sqrt{7}}\tan^{-1}\left(\frac{4}{\sqrt{7}}(x+\frac{1}{4})\right) + C$

$= \ln\left|\frac{x}{\sqrt{2x^2+x+1}}\right| - \frac{\sqrt{7}}{7}\tan^{-1}\left(\frac{4}{\sqrt{7}}(x+\frac{1}{4})\right) + C$

2.) $\int_t^3 \frac{1}{(x-2)^{3/2}}dx = \left[-2(x-2)^{-\frac{1}{2}}\right]_t^3$

$= -2 + 2(t-2)^{-1/2}$

$\lim_{t\to 2^+}\left(-2+2(t-2)^{-1/2}\right) = +\infty$

Thus, J diverges.

3.) $f(x) = \frac{1}{2}\left(\frac{3}{2}e^{\frac{2x}{3}} + \frac{3}{2}e^{-\frac{2x}{3}}\right)$

$f'(x) = \frac{1}{2}\left(e^{\frac{2}{3}x} - e^{-\frac{2}{3}x}\right)$

$1+f'(x)^2 = 1 + \frac{1}{4}\left(e^{\frac{4x}{3}} - 2 + e^{-\frac{4}{3}x}\right)$

$= \frac{e^{\frac{4x}{3}}}{4} + \frac{1}{2} + \frac{e^{-\frac{4}{3}x}}{4}$

$= \frac{1}{4}\left(e^{\frac{2x}{3}} + e^{-\frac{2}{3}x}\right)^2$

$L = \int_0^1 \frac{1}{2}\left(e^{\frac{2x}{3}} + e^{-\frac{2}{3}x}\right)dx$

$= \frac{1}{2}\left[\frac{3}{2}e^{\frac{2}{3}x} - \frac{3}{2}e^{-\frac{2}{3}x}\right]_0^1$

$= \frac{1}{2}\left[\frac{3}{2}e^{\frac{2}{3}} - \frac{3}{2}e^{-\frac{2}{3}}\right]$

$= \frac{3}{4}\left(e^{\frac{2}{3}} - e^{-\frac{2}{3}}\right).$