MATH 102.1 (Term 161)
Quiz 3 (Sects. 7.1, 7.2 & 7.3) **Duration: 20min**

Name: **ID number:**

1.)(3pts)Evaluate the integral $I = \int \cos^4 x dx$.
2.)(4pts)Evaluate the integral $J = \int \frac{1}{x^2\sqrt{4x^2-25}} dx$.
3.)(3pts)Evaluate the integral $K = \int (1+x^2)\sin x dx$.

1.) $\cos^2 x = \frac{1+\cos 2x}{2}$

$$I = \int \left(\frac{1+\cos 2x}{2}\right)^2 dx$$

$$= \frac{1}{4}\int (1 + 2\cos 2x + \cos^2 2x)\, dx$$

$$= \frac{1}{4}\int \left(1 + 2\cos 2x + \frac{1+\cos 4x}{2}\right) dx$$

$$= \frac{1}{4}\int \left(\frac{3}{2} + 2\cos 2x + \frac{\cos 4x}{2}\right) dx$$

$$= \frac{3}{8}x + \frac{\sin 2x}{4} + \frac{\sin 4x}{32} + C$$

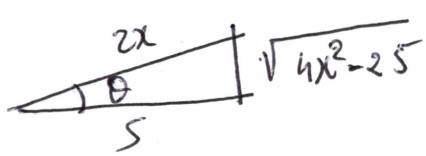
2.) $$J = \int \frac{dx}{x^2\sqrt{4x^2-25}}$$

$2x = 5\sec\theta, \quad 0 \le \theta < \pi/2$
$2dx = 5\sec\theta\tan\theta\, d\theta$

$$J = \int \frac{5/2\ \sec\theta\tan\theta\, d\theta}{\frac{25}{4}\sec^2\theta\ 5\tan\theta}$$

$$= \frac{2}{25}\int \cos\theta\, d\theta$$

$$= \frac{2}{25}\sin\theta + C$$



$$J = \frac{2}{25}\frac{\sqrt{4x^2-25}}{2x} + C$$

3.) Integration by parts.

$u = 1+x^2 \rightarrow u' = 2x$
$v' = \sin x \rightarrow v = -\cos x$

$$K = -(1+x^2)\cos x + \int 2x\cos x\, dx$$

$u = 2x \rightarrow u' = 2$
$v' = \cos x \rightarrow v = \sin x$

$$K = -(1+x^2)\cos x + 2x\sin x - 2\int \sin x\, dx$$

$$= -(1+x^2)\cos x + 2x\sin x + 2\cos x + C$$

MATH 102.3 (Term 161)
Quiz 3 (Sects. 7.1, 7.2 & 7.3) **Duration: 20min**

Name: **ID number:**

1.)(3pts)Evaluate the integral $I = \int \sin^4 x dx$.
2.)(4pts)Evaluate the integral $J = \int \frac{1}{x^2\sqrt{9x^2+16}} dx$.
3.)(3pts)Evaluate the integral $K = \int (1-x^2)e^{-x} dx$.

1.) $\sin^2 x = \frac{1-\cos 2x}{2}$

$$I = \int \left(\frac{1-\cos 2x}{2}\right)^2 dx$$

$$= \frac{1}{4}\int \left(1 - 2\cos 2x + \cos^2 2x\right) dx$$

$$= \frac{1}{4}\int \left(1 - 2\cos 2x + \frac{1+\cos 4x}{2}\right) dx$$

$$= \frac{1}{4}\int \left(\frac{3}{2} - 2\cos 2x + \frac{\cos 4x}{2}\right) dx$$

$$= \frac{3}{8}x - \frac{\sin 2x}{4} + \frac{\sin 4x}{32} + C$$

2.) $J = \int \frac{dx}{x^2\sqrt{9x^2+16}}$

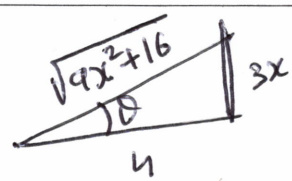
$3x = 4\tan\theta, \quad -\frac{\pi}{2} < \theta < \pi/2$

$3dx = 4\sec^2\theta \, d\theta$

$$J = \int \frac{\frac{4}{3}\sec^2\theta \, d\theta}{\frac{16}{9}\tan^2\theta \cdot 4\sec\theta}$$

$$= \frac{3}{16}\int \frac{\cos\theta}{\sin^2\theta} d\theta$$

$$= -\frac{3}{16}\frac{1}{\sin\theta} + C$$



$$\sin\theta = \frac{3x}{\sqrt{9x^2+16}}$$

$$J = -\frac{3}{16}\frac{\sqrt{9x^2+16}}{3x} + C$$

3.) Integration by parts

$u = 1-x^2 \longrightarrow u' = -2x$

$v' = e^{-x} \longrightarrow v = -e^{-x}$

$$K = -(1-x^2)e^{-x} - \int 2x e^{-x} dx$$

$u = 2x \longrightarrow u' = 2$

$v' = -e^{-x} \longrightarrow v = e^{-x}$

$$K = -(1-x^2)e^{-x} + 2xe^{-x} - 2\int e^{-x} dx$$

$$= -(1-x^2)e^{-x} + 2xe^{-x} + 2e^{-x}$$

$$= (x^2+2x+1)e^{-x}$$