MATH 102.1 (Term 161)
Quiz 2 (Sects. 6.1 & 6.2) **Duration: 20min**

Name: **ID number:**

1.)(5pts)Write the integral given the area of the region bounded by the four lines $y = x$, $y = x - 2$, $y = -x - 1$ and $y = \frac{1}{5}(-x + 3)$ (do not evaluate the integral).
2.)(5pts)Find the volume of the solid generated by revolving the region bounded by the curves $y = x^2$ and $y = \sqrt{x}$ about the line $x = -1$.

1.) $L_1: y = x$; $L_3: y = x - 2$

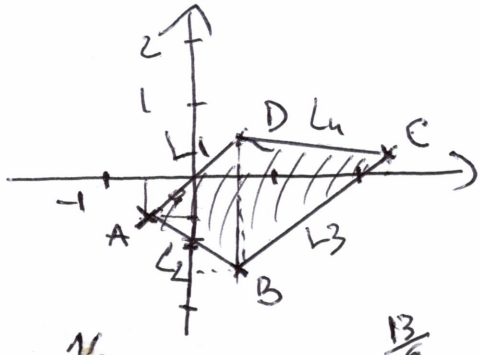
$L_2: y = -x - 1$; $L_4: y = \frac{1}{5}(-x + 3)$

$L_1 \cap L_2 = \{A\}$, $x = -x - 1$, $x = -\frac{1}{2}$, $A\begin{pmatrix} -1/2 \\ -1/2 \end{pmatrix}$

$L_3 \cap L_2 = \{B\}$, $x - 2 = -x - 1$, $x = \frac{1}{2}$, $B\begin{pmatrix} 1/2 \\ -3/2 \end{pmatrix}$

$L \cap L_4 = \{C\}$, $x - 2 = \frac{1}{5}(-x + 3)$, $x = \frac{13}{6}$, $C\begin{pmatrix} 13/6 \\ 1/6 \end{pmatrix}$

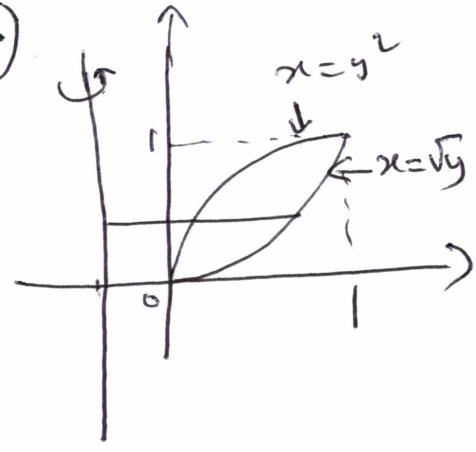
$L_1 \cap L_4 = \{D\}$; $x = \frac{1}{5}(-x + 3)$, $x = \frac{1}{2}$, $D\begin{pmatrix} 1/2 \\ 1/2 \end{pmatrix}$



$$\text{Area}(R) = \int_{-1/2}^{1/2} \left[x - (-x-1)\right] dx + \int_{1/2}^{13/6} \left[\frac{-x}{5} + \frac{3}{5} - (x-2)\right] dx$$

$$= \int_{-1/2}^{1/2} (2x+1)\, dx + \int_{1/2}^{13/6} \left(\frac{-6}{5}x + \frac{13}{5}\right) dx$$

2.)



$$V = \int_0^1 \pi\left[(\sqrt{y}+1)^2 - (y^2+1)^2\right] dy$$

$$= \int_0^1 \pi\left(y + 2\sqrt{y} - y^4 - 2y^2\right) dy$$

$$= \pi\left[\frac{y^2}{2} + \frac{4}{3}y^{3/2} - \frac{y^5}{5} - \frac{2y^3}{3}\right]_0^1$$

$$= \frac{29\pi}{30}$$

MATH 102.3 (Term 161)
Quiz 2 (Sects. 6.1 & 6.2) **Duration: 20min**

Name: **ID number:**

1.)(5pts)Write the integral given the area of the region bounded by the four lines $y = x$, $y = x - 2$, $y = -x - 1$ and $y = -\frac{7}{3}x + 3$ (do not evaluate the integral).
2.)(5pts)Find the volume of the solid generated by revolving the region bounded by the curves $y = x^2$ and $y = \sqrt{x}$ about the line $y = -1$.

1) $L_1: y = x$; $L_3: y = x - 2$

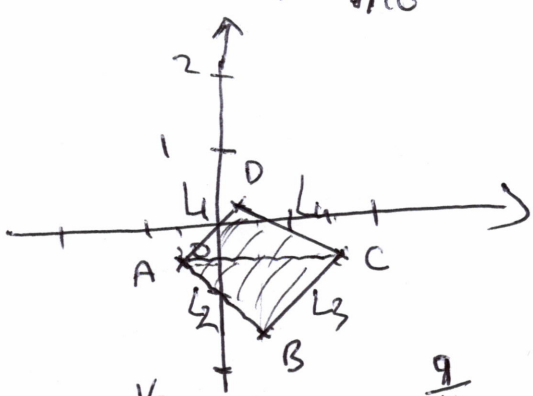
$L_2: y = -x - 1$; $L_4: y = -\frac{7}{3}x + 3$

$L_4 \cap L_2 = \{A\}$, $x = -x - 1$, $x = -\frac{1}{2}$, $A\begin{pmatrix} -1/2 \\ -1/2 \end{pmatrix}$

$L_2 \cap L_3 = \{B\}$, $-x - 1 = x - 2$, $x = \frac{1}{2}$, $B\begin{pmatrix} 1/2 \\ -3/2 \end{pmatrix}$

$L_3 \cap L_4 = \{C\}$; $x - 2 = -\frac{7}{3}x + 3$, $x = \frac{3}{2}$, $C\begin{pmatrix} 3/2 \\ -1/2 \end{pmatrix}$

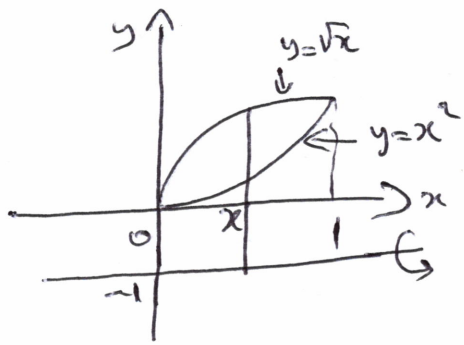
$L_4 \cap L_1 = \{D\}$, $x = -\frac{7}{3}x + 3$, $x = 9/10$, $D\begin{pmatrix} 9/10 \\ 9/10 \end{pmatrix}$



$$\text{Area}(R) = \int_{-\frac{3}{2}}^{-1/2} \left[(y+2) - (-y-1)\right] dx + \int_{-1/2}^{\frac{9}{10}} \left[-\frac{3}{7}(y-3) - y\right] dy$$

$$= \int_{-\frac{3}{2}}^{-1/2} (2y+3)\, dy + \int_{-\frac{1}{2}}^{\frac{9}{10}} \left(\frac{-10}{7}y + \frac{9}{7}\right) dy$$

2)



$$V = \int_0^1 \pi\left[(\sqrt{x}+1)^2 - (x^2+1)^2\right] dx$$

$$= \pi \int_0^1 \left(x + 2\sqrt{x} - x^4 - 2x^2\right) dx$$

$$= \pi\left[\frac{x^2}{2} + \frac{4}{3}x^{3/2} - \frac{1}{5}x^5 - \frac{2x^3}{3}\right]_0^1$$

$$= \frac{29}{30}\pi$$