MATH 102.1 (Term 161)

Quiz 1 (Sects. 5.3, 5.4 & 5.5) Duration: 20min

Name: **ID number:**

1.)(4pts)Evaluate $\lim_{n\to\infty}\sum_{i=1}^{n} x_i^2(x_i^3+1)^6\Delta x$ **by expressing this limit as a definite integral on the interval** $[0,1]$.

2.)(6pts)Evaluate $A=\int_2^3 (x-2)^8(x+3)\,dx$, $B=\int_{-1}^{0} x^9\sqrt[4]{x^5+1}dx$, $C=\int_{\pi/4}^{\pi/3}\frac{\sin 2x}{(1+\cos^2 x)^5}dx$

1.)

$$\lim_{n\to\infty}\sum_{i=1}^{n} x_i^2(x_i^3+1)^6\Delta x = \int_0^1 x^2(x^3+1)^6dx$$

$u=x^3+1$, $du=3x^2dx$

$$\int_0^1 x^2(x^3+1)^6dx=\frac{1}{3}\int_1^2 u^6du=\frac{1}{21}\left[u^7\right]_1^2$$

$$=\frac{127}{21}$$

2.) $A=\int_2^3 (x-2)^8(x+3)\,dx$

$u=x-2$, $du=dx$

$$A=\int_0^1 u^8(u+5)\,du=\int_0^1(u^9+5u^8)du$$

$$=\left[\frac{u^{10}}{10}+\frac{5u^9}{9}\right]_0^1=\frac{1}{10}+\frac{5}{9}=\frac{59}{90}$$

$$B=\int_{-1}^0 x^9\sqrt[4]{x^5+1}\,dx$$

$u=x^5+1$, $du=5x^4dx$

$$B=\frac{1}{5}\int_0^1 u^{\frac{1}{4}}(u-1)\,du$$

$$=\frac{1}{5}\int_0^1\left(u^{\frac{5}{4}}-u^{1/4}\right)du$$

$$=\frac{1}{5}\left[\frac{4}{9}u^{\frac{9}{4}}-\frac{4}{5}u^{5/4}\right]_0^1$$

$$=\frac{1}{5}\left(\frac{4}{9}-\frac{4}{5}\right)=\frac{-16}{225}$$

$$C=\int_{\pi/4}^{\pi/3}\frac{\sin 2x}{(1+\cos^2 x)^5}dx$$

$u=1+\cos^2 x$, $du=-2\cos x\sin x\,dx$
$=-\sin 2x\,dx$

$$C=-\int_{3/2}^{5/4}\frac{du}{u^5}=-\int_{3/2}^{5/4}u^{-5}du$$

$$=\left[\frac{u^{-4}}{4}\right]_{3/2}^{5/4}$$

$$=\frac{1}{4}\left(\left(\frac{5}{4}\right)^{-4}-\left(\frac{3}{2}\right)^{-4}\right)$$

MATH 102.3 (Term 161)

Quiz 1 (Sects. 5.3, 5.4 & 5.5) Duration: 20min

Name: ID number:

1.)(4pts)Evaluate $\lim_{n\to\infty}\sum_{i=1}^{n} x_i^3(x_i^4+1)^9\Delta x$ by expressing this limit as a definite integral on the interval $[-1,0]$.

2.)(6pts)Evaluate $A=\int_{-1}^{0}(x+1)^9(x-1)^2dx$, $B=\int_{-1}^{0}x^{13}\sqrt[5]{x^7+1}\,dx$, $C=\int_{\pi/6}^{\pi/4}\frac{\sin 2x}{(2+\sin^2 x)^4}dx$

1.)

$$\lim_{n\to\infty}\sum_{i=1}^{n} x_i^3(x_i^4+1)^9\Delta x=\int_{-1}^{0}x^3(x^4+1)^9dx$$

$u=x^4+1,\quad du=4x^3dx$

$$\int_{-1}^{0}x^3(x^4+1)^9dx=\frac{1}{4}\int_{2}^{1}u^9du=\frac{1}{40}\left[u^{10}\right]_2^1$$

$$=-\frac{1023}{40}$$

2.) $A=\int_{-1}^{0}(x+1)^9(x-1)\,dx$

$u=x+1,\quad du=dx,$

$$A=\int_0^1 u^9(u-2)\,du=\int_0^1(u^{10}-2u^9)\,du$$

$$=\left[\frac{1}{11}u^{11}-\frac{u^{10}}{5}\right]_0^1=\frac{1}{11}-\frac{1}{5}$$

$$=\frac{-6}{55}$$

$$B=\int_{-1}^{0}x^{13}\sqrt[5]{x^7+1}\,dx$$

$u=x^7+1,\quad du=7x^6dx$

$$B=\int_0^1 u^{\frac{1}{5}}(u-1)\frac{du}{7}$$

$$=\frac{1}{7}\int_0^1\left(u^{\frac{6}{5}}-u^{1/5}\right)du$$

$$=\frac{1}{7}\left[\frac{5}{11}u^{\frac{11}{5}}-\frac{5}{6}u^{\frac{6}{5}}\right]_0^1=-\frac{1}{7}\,\frac{25}{66}$$

$$C=\int_{\pi/6}^{\pi/4}\frac{\sin 2x}{(2+\sin^2x)^4}dx$$

$u=2+\sin^2x,\quad du=2\sin x\cos x\,dx$

$=\sin 2x\,dx$

$$C=\int_{\frac{9}{4}}^{\frac{5}{2}}\frac{1\,du}{u^4}=\int_{9/4}^{5/2}u^{-4}du=\left[-\frac{u^{-3}}{3}\right]_{9/4}^{\frac{5}{2}}$$

$$=\frac{1}{3}\left(\left(\frac{9}{4}\right)^{-3}-\left(\frac{5}{2}\right)^{-3}\right)$$