King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
**Math 101(22&31) Class Test 2 Fall 2016(161)**

ID#:_____ NAME:_____

(1) Sketch the graph of a function f that satisfies all of the given conditions:

$$\lim_{x\to -4^-} f(x) = -\infty;\ \lim_{x\to -4^+} f(x) = \infty;$$
$$\lim_{x\to -\infty} f(x) = 0;\ \lim_{x\to -2} f(x) = 1;$$
$$f \text{ is undefined at } -2;\ \lim_{x\to 1^-} f(x) = -1;$$
$$\lim_{x\to 1^+} f(x) = 2;\ f(1) = 1$$

(2) Evaluate the limit, if it exists:

(a) $\lim_{x \to 3^-} \frac{|x^2-9|}{x-3}$

(b) $\lim_{x \to 1} \frac{4-x^2}{2-x-x^2}$

(c) $\lim_{x \to -2} [[\frac{1}{2}x + 1]]$, where $[[.]]$ denotes the greatest integer function.

(d) $\lim_{x\to+\infty} \frac{2x+x\cos x}{5x^2-2x+1}$.

(e) $\lim_{x\to 0^+} x\sin(\frac{\sqrt{x+2}}{x})$.

(e) $\lim_{x\to-\infty} (-33x+1)^3(2x-1)^2x$

(3) Find the horizontal asymptotes of the graph of the function $f(x) = \tan^{-1}(\frac{\sqrt{9x^2+2}}{3x+7})$.

(4) Use the graph of $f(x) = 2\sqrt{x}$ to find a number $\delta$ such that $|2\sqrt{x} - 4| < 1$ whenever $|x - 4| < \delta$. (Show your work and write your answer in simplest rational form $\frac{p}{q}$).

(5) Use the Intermediate Value Theorem to show that there is a root of the equation $e^{-x^2} = x$ between $0$ and $1$.

(6)Find the values of $a$ and $b$ that make the function

$$f(x) = \begin{cases} 3 & \text{if } x = 1 \\ ax^2 - bx + 3 & \text{if } 1 < x < 2 \\ 2x - a + b & \text{if } 2 \leq x < 3 \\ 6 & \text{if } x = 3. \end{cases}$$

continuous on the closed interval $[1, 3]$. (Use limits to justify your steps)

(7) Given the function $f(x) = \frac{2x^2+kx-14}{x-2}$, where $k$ is a constant, find $k$ such that $x = 2$ is a removable discontinuity of $f$. (Give reasons to your steps).

(8) The displacement (in meters) of a particle moving in a straight line is given by $s = \frac{1}{\sqrt{5-t}}$ where $t$ is measured in seconds. Use limits to find the instantaneous velocity of the particle when $t = 1$.

(9) Prove that $\lim_{x \to 1} 2x + 2 = 4$.

(10) Find the equation of the tangent line to $f(x) = x - \frac{1}{x}$ at $x = 3$.

(11) Show that $f(x) = \sqrt{16 - x}$ is continuous on the interval $(-\infty, 16]$.

(12) Let

$$f(x) = \begin{cases} x^2 & \text{if } x \leq 2 \\ mx + r & \text{if } x > 2. \end{cases}$$

Find the values of $m$ and $r$ that make $f$ differentiable everywhere.