1. If $f(x) = e^x g(x)$, $g(0) = 2$ and $g'(0) = 5$, then $f'(0) =$

a) 7
b) 10
c) 6
d) 8
e) 11

$$f'(x) = e^x g(x) + e^x g'(x)$$

$$\therefore f'(0) = g(0) + g'(0)$$

$$= 2 + 5$$

$$= 7$$

2. If $f(t) = \csc(t)$, then $f''\left(\frac{\pi}{6}\right) =$

a) 14
b) 20
c) 10
d) -5
e) 0

$$f'(t) = -\csc t \cot t$$

$$f''(t) = \csc^3 t + \csc t \cot^2 t$$

$$= (2)^3 + 2\left(\sqrt{3}\right)^2$$

$$= 8 + 6 = 14$$

3. If $g(x) = \frac{h(x)}{x}$, $h(2) = 4$, $h'(2) = -3$, then the slope of the normal line to the curve $g(x)$ at $x = 2$ is

a) $\frac{2}{5}$
b) $\frac{3}{5}$
c) $\frac{1}{5}$
d) $\frac{2}{3}$
e) $\frac{-3}{2}$

$$g'(x) = \frac{x\,h'(x) - h(x)}{x^2}$$

$$\therefore g'(2) = \frac{2h'(2) - h(2)}{4} = \frac{-6-4}{4} = -\frac{10}{4} = -\frac{5}{2}$$

$$\therefore m_{N} = \frac{-1}{g'(2)} = \frac{2}{5}.$$

4. $\lim\limits_{x\to 1} \frac{\sin(x-1)}{x^2+x-2} =$

a) $\frac{1}{3}$
b) $-\frac{1}{3}$
c) 1
d) -1
e) 0

$$\lim_{x\to 1} \frac{\sin(x-1)}{(x-1)(x+2)}$$

$$\lim_{x\to 1} \frac{\sin(x-1)}{x-1} = 1,$$

$$\lim_{x\to 1} \frac{1}{x+2} = \frac{1}{3}$$

$$\therefore \lim_{x\to 1} \frac{\sin(x-1)}{(x-1)(x+2)} = \frac{1}{3}$$

5. If $f(x) = \alpha x + \beta$ is the linearization of $f(x) = e^{-\sin(4x)}$ at $x = 0$, then the sum $\alpha + \beta$ equals

a) -3
b) -2
c) -1
d) 0
e) 1

$$f'(x) = -4\cos(4x)\, e^{-\sin(4x)}$$

$$\Rightarrow f'(0) = -4$$

$$L(x) = f(0) + f'(0)(x-0)$$

$$= 1 - 4x$$

$$\therefore \alpha + \beta = -4 + 1 = -3$$

6. The position of a particle is given by the equation

$$s(t) = 2t^3 - 9t^2 + 12t$$

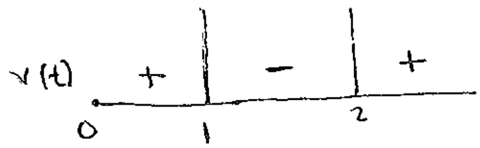
where t is measured in seconds and S in meters. The total distance traveled by the particle during the first 3 seconds is

a) $11\,m$
b) $13\,m$
c) $15\,m$
d) $17\,m$
e) $19\,m$

$$v(t) = s'(t) = 6t^2 - 18t + 12$$

$$= 6(t^2 - 3t + 2)$$

$$= 6(t-1)(t-2)$$



$$\therefore d = |s(1) - s(0)| + |s(2) - s(1)| + |s(3) - s(2)|$$

$$= 5 + |4 - 5| + |9 - 4|$$

$$= 5 + 1 + 5 = 11\,m.$$

7. If $y = \ln(\cos(\ln x))$, then $\frac{dy}{dx} =$

a) $\frac{-\tan(\ln x)}{x}$
b) $-\tan(\ln x)$
c) $\frac{\tan(hx)}{x}$
d) $\tan(\ln x)$
e) $\frac{\tan(\ln x)}{x^2}$

$$\frac{dy}{dx} = \frac{1}{\cos(\ln x)} \cdot \left(-\sin(\ln x)\right) \cdot \frac{1}{x}$$

$$= -\frac{\tan(\ln x)}{x}.$$

8. If $y = e^{e^x}$, then $y''(x) =$

a) $e^{e^x+x}(e^x+1)$
b) $e^{e^x-x}(e^x-1)$
c) $e^{e^x+x}(e^x+2)$
d) $e^{e^x-x}(e^x-2)$
e) $e^{e^x+x}(e^x+3)$

$$y' = e^{e^x} e^x$$

$$y'' = e^{e^x} \cdot e^x \cdot e^x + e^{e^x} e^x$$

$$\Rightarrow y'' = e^{e^x} e^x (e^x+1)$$

$$= e^{e^x+x}(e^x+1)$$

9. For $f(x) = \dfrac{(x+1)(x+4)(x+8)\,e^{x^2}}{\sqrt{x+2}}$, $f'(0) =$
(**Hint: You may use logarithmic differentiation**)

a) $18\sqrt{2}$
b) $20\sqrt{2}$
c) $30\sqrt{2}$
d) $34\sqrt{2}$
e) $40\sqrt{2}$

$$\ln f(x) = \ln(x+1) + \ln(x+4) + \ln(x+8) + x^2 - \tfrac{1}{2}\ln(x+2)$$

$$\Rightarrow \frac{f'(x)}{f(x)} = \frac{1}{x+1} + \frac{1}{x+4} + \frac{1}{x+8} + 2x - \frac{1}{2(x+2)}$$

$$f(0) = 16\sqrt{2}$$

$$\Rightarrow f'(0) = 16\sqrt{2}\left(1 + \cancel{\tfrac{1}{4}} + \tfrac{1}{8} - \cancel{\tfrac{1}{4}}\right) = 16\sqrt{2}\cdot\frac{9}{8}$$

$$= 18\sqrt{2}$$

10. The radius of a circle decreased from $2\,cm$ to $1.9\,cm$. Use differentials to estimate the percentage error in calculating its area.

a) decreased by 10%
b) decreased by 20%
c) decreased by 5%
d) increased by 20%
e) increased by 5%

$$A = \pi r^2, \quad dr = -0.1, \quad r = 2$$

$$\frac{dA}{A} \times 100 = \frac{2\cancel{\pi}\cancel{r}\,dr}{\cancel{\pi}r^{\cancel{2}}}(100)$$

$$= \frac{\cancel{2}(-0.1)(100)}{\cancel{2}}\ \%$$

$$= -10\ \%$$

11. $\lim_{x\to 0}(1+x)^{\frac{1}{x\ln 5}} =$

a) $e^{\frac{1}{\ln 5}}$

b) $\ln 5$

c) e

d) 5

e) $\dfrac{1}{\ln 5}$

$$\lim_{x\to 0}\left(\left(1+x\right)^{\frac{1}{x}}\right)^{\frac{1}{\ln 5}}$$

$$= \left(\lim_{x\to 0}\left(1+x\right)^{\frac{1}{x}}\right)^{\frac{1}{\ln 5}}$$

$$= e^{\frac{1}{\ln 5}}$$

12. Find y'' if $x^3 + y^3 = 1$.

a) $y'' = \dfrac{-2x}{y^5}$

b) $y'' = \dfrac{-2xy^3+1}{y^4}$

c) $y'' = \dfrac{x^2}{y^5}$

d) $y'' = \dfrac{2x^2-2x}{y^3}$

e) $y'' = \dfrac{x^2+y^2}{y^4}$

$$3x^2 + 3y^2 y' = 0$$

$$\Rightarrow y' = -\frac{x^2}{y^2}$$

$$\Rightarrow y'' = \frac{-2xy^2 + x^2(2yy')}{y^4}$$

$$= \frac{-2xy^2 + 2x^2 y\left(\frac{-x^2}{y^2}\right)}{y^4}$$

$$= \frac{-2xy^4 - 2yx^4}{y^6}$$

$$= \frac{-2xy\left(y^3 + x^3\right)}{y^6}$$

$$= \frac{-2x}{y^5}$$

13. The slope of the tangent line to the curve $y = \dfrac{\sin^{-1} x}{\cos^{-1} x}$ at $x = 0$ is

a) $\frac{2}{\pi}$
b) $\frac{\pi}{2}$
c) 1
d) 0
e) $\frac{-\pi}{2}$

$$y' = \frac{\cos^{-1}x\left(\frac{1}{\sqrt{1-x^2}}\right) - \sin^{-1}x\left(\frac{-1}{\sqrt{1-x^2}}\right)}{\left(\cos^{-1}x\right)^2}$$

$$y'\Big|_{x=0} = \frac{\frac{\pi}{2}(1) - 0}{\frac{\pi^2}{4}} = \frac{\pi}{2}\cdot\frac{4}{\pi^2} = \frac{2}{\pi}$$

14. The slope of the tangent line to the curve of $\tan^{-1}(xy) = \frac{\pi}{8}(x^2 + y^2)$ at $(1, 1)$ is

a) -1
b) 1
c) $\frac{\pi}{4}$
d) $\frac{\pi}{8}$
e) $\frac{-\pi}{4}$

$$\frac{xy' + y}{1 + x^2y^2} = \frac{\pi}{8}(2x + 2yy')$$

let $y'\Big|_{(1,1)} = m$

$$\Rightarrow \frac{m+1}{2} = \frac{\pi}{8}(2 + 2m)$$

$$\Rightarrow 4m + 4 = 2\pi + 2\pi m$$

$$\Rightarrow (4 - 2\pi)m = 2\pi - 4$$

$$\Rightarrow m = \frac{2\pi - 4}{4 - 2\pi} = -1$$

15. $\lim\limits_{x\to 1} \dfrac{\ln(x^2+e-1)-1}{x-1} = \dfrac{d}{dx}\left(\ln(x^2+e-1)\right)\Big|_{x=1}$

a) $\frac{2}{e}$
b) 2
c) e
d) 1
e) 3

$= \dfrac{2x}{x^2+e-1}\Big|_{x=1}$

$= \dfrac{2}{e}$

16. The number of points on the graph of the function $f(x) = \cos^3 x - 3\sin x$, $0 \le x \le 2\pi$ at which the tangent line is horizontal is:

a) 2
b) 1
c) 0
d) 3
e) 4

$f'(x) = 3\cos^2 x\,(-\sin x) - 3\cos x$

$= 3\cos x\,(-\cos x \sin x - 1)$

$\Rightarrow \cos x = 0$ or $\cos x \sin x = -1$

$\cos x = 0 \Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$.

$\cos x \sin x = -1 \Rightarrow \sin(2x) = -2$ impossible.

So, the graph of $f(x)$ has two points at which the tangent is horizontal.

17. If $y = (2x + \ln x)^{\sin(x-1)}$, then $y'(1) =$

a) $\ln 2$

b) 0

c) 1

d) 2

e) $\frac{3}{2}$

$$\ln y = \sin(x-1)\ln(2x+\ln x)$$

$$\frac{y'}{y} = \cos(x-1)\ln(2x+\ln x) + \sin(x-1)\cdot\frac{2+\frac{1}{x}}{2x+\ln x}$$

$$y'(1) = (1)\ln 2 + 0 = \ln 2$$

Note: $y(1) = 1$.

18. Gas is escaping from a spherical ballon at a rate of 2 cubic feet per minute. How fast is the surface area shrinking when the radius of the ballon is $12\,feet$?

a) $-\frac{1}{3}\,ft^2/min$

b) $-\frac{1}{2\pi}\,ft^2/min$

c) $-\frac{1}{4\pi}\,ft^2/min$

d) $-1\,ft^2/min$

e) $1\,ft^2/min$

$$V = \frac{4}{3}\pi r^3, \quad \frac{dV}{dt} = -2\ ft^3/min$$

$$\frac{dV}{dt} = 4\pi r^2\frac{dr}{dt}$$

$$r = 12 \Rightarrow -2 = 4\pi(144)\frac{dr}{dt}$$

$$\Rightarrow \frac{dr}{dt} = \frac{-1}{288\pi}\ ft/min.$$

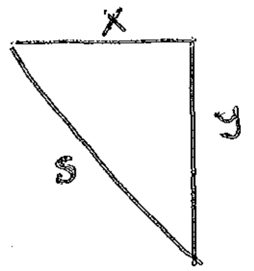
$$S = 4\pi r^2$$

$$\Rightarrow \frac{dS}{dt} = 8\pi r\cdot\frac{dr}{dt}$$

$$\Rightarrow \left.\frac{dS}{dt}\right|_{r=12} = 8\pi(12)\left(\frac{-1}{288\pi}\right)$$

$$= \frac{-1}{3}\ ft^2/min$$

19. Two objects start moving from the same point. One travels south at $2\,m/s$ and the other travels west at $4\,m/s$. At what rate is the distance between the objects increasing $5\,s$ later?



a) $2\sqrt{5}\,m/s$
b) $\frac{\sqrt{5}}{2}\,m/s$
c) $4\sqrt{5}\,m/s$
d) $\frac{\sqrt{5}}{2}\,m/s$
e) $5\,m/s$

$$S^2 = x^2 + y^2$$

$$2S\frac{dS}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$$

after 5s : $x = 20,\ y = 10 \Rightarrow S = 10\sqrt{5}$

$$\Rightarrow 10\sqrt{5}\,\frac{dS}{dt} = (20)(4) + (10)(2) = 100$$

$$\Rightarrow \frac{dS}{dt} = \frac{10}{\sqrt{5}} = \frac{10\sqrt{5}}{5} = 2\sqrt{5}\ m/s$$

20. If $f(x) = x^{36} + \sin(2x)$, then $f^{(37)}(x) =$

a) $2^{37}\cos(2x)$
b) $2^{37}\sin(2x)$
c) $-2^{37}\sin(2x)$
d) $-2^{37}\cos(2x)$
e) $\cos(2x)$

$$\frac{d^{37}}{dx^{37}}\left(x^{36}\right) = 0\ ,$$

let $g(x) = \sin(2x)$

$g'(x) = 2\cos(2x)$

$g''(x) = -2^2\sin(2x)$

$g'''(x) = -2^3\cos(2x)$

$g''''(x) = +2^4\sin(2x)$

$$\therefore\ f^{(37)}(x) = 0 + 2^{37}\cos(2x)$$

$$= 2^{37}\cos(2x)$$