Dept of Mathematics and Statistics
King Fahd University of Petroleum & Minerals

AS482: Actuarial Contingencies II
Dr. Mohammad H. Omar
FINAL Exam Term 161 FORM A
Sunday Jan 15 2017
7.00pm-9.30pm

Name_____ ID#:_____ Serial #:___

Instructions.

1. Please turn off your cell phones and place them under your chair. Any student caught with mobile phones on, with any cheating material or cheating actions during the exam will be considered under the **cheating rules** of the University.
2. If you need to leave the room, please do so quietly so not to disturb others taking the test. No two person can leave the room at the same time. No extra time will be provided for the time missed outside the classroom.
3. Only materials provided by the instructor can be present on the table during the exam.
4. Do not spend too much time on any one question. If a question seems too difficult, leave it and go on.
5. Use the blank portions of each page for your work. Extra blank pages can be provided if necessary. If you use an extra page, indicate clearly what problem you are working on.
6. Only answers supported by work will be considered. Unsupported guesses will not be graded.
7. While every attempt is made to avoid defective questions, sometimes they do occur. In the rare event that you believe a question is defective, the instructor cannot give you any guidance beyond these instructions.
8. Mobile calculators, I-pad, or communicable devices are disallowed. Use regular scientific calculators or financail calculators only. Write important steps to arrive at the solution of the following problems.

The test is 150 minutes, GOOD LUCK, and you may begin now!

Question	Total Marks	Marks Obtained	Comments
1	6+6+4=16		
2	2+3+4=9		
3	3+17=20		
4	4+5=9		
5	5+3+2=10		
6	10		
7	6+4+3+3=16		
8	1+4=5		
9	1+4=5		
Total	100		

Extra blank page

1. (6+6+4=16 points) Given the HRF $\mu_{x+t}^{(1)} = \dfrac{1}{75-t}$, for $0 \le t < 75$, and $\mu_{x+t}^{(2)} = \dfrac{2}{50-t}$, for $0 \le t < 50$, compute each of the following:

(a) ${}_tp_x'^{(1)}$, ${}_tp_x'^{(2)}$ and ${}_tp_x^{(\tau)}$
(b) $q_x^{(1)}, q_x^{(2)}$ and $q_x^{(\tau)}$
(c) $q_x'^{(1)}$and $q_x'^{(2)}$

2. (2+3+4=9 points) A large corporation would like to *self-insure* its employees' annual dental expenses. The company would like to maintain sufficient assets to be reasonably sure of paying the first $\$2,000,000$ in expenses each year and will reinsure aggregate losses above that amount. All assets are to be invested in *one-year corporate bonds* yielding 6% annual interest. Bonds may be purchased in increments of \$1000. Assuming the portfolio earns the 6% total return during the year, answer the following questions:

(a) What **dollar amount of bonds** now will be sufficient to pay the first $\$2,000,000$ of claims?

(b) Each bond has a 3% chance of default during the year. If a bond defaults, the bond's value at year end will be uniformly distributed between \$0 and \$1000. In this case, to have an expected value of *at least* $\$2,000,000$ portfolio value at year end, what **number of bonds** should be purchased?

(c) Suppose the corporation purchases the bonds suggested in (b) above. Since the value of each bond at year end is uncertain, the corporation's assets may exceed or fall short of the $\$2,000,000$ needed at year end. For planning purposes, the corporation simulated the return on each bonds and obtained the following after 20 runs of portfolio simulations:

Run No.	**Simulated Number Defaults**	**Simulated Year-end Portfolio Value**
1	71	2007013
2	83	1996821
3	76	1999596
4	88	1999620
5	74	2004403
6	101	1985824
7	88	1996517
8	66	2006397
9	81	2001682
10	84	1998392

Run No.	**Simulated Number Defaults**	**Simulated Year-end Portfolio Value**
11	70	2009449
12	69	2006524
13	76	2000335
14	74	2001262
15	86	1993844
16	86	1996388
17	71	2005363
18	84	2000393
19	83	1996413
20	63	2004851

Using these 20 runs, estimate
i) the **probability** that the insurer has funds *in excess of* \$2,000,000 at year end.and
ii) the **20th percentile** of the fund value at year end.

3. (3+17=20 points) An employer establishes a *Defined Contribution* (DC) pension plan. Since there is no vesting period, on withdrawal from the plan for any reason before normal retirement age of 65, the proceeds of the invested contributions are paid to the employee or the employee's survivors. The contribution rate is set using the following assumptions.

(i) The *salary rate* function is given by $s_y = 1.04^y$ and salaries are assumed to *increase continuously.*
(ii) Month-end *monthly contributions* are payable at a fixed percentage, c, of the salary rate at time t.
(iii) Contributions are assumed to earn investment *returns* of 10% per year.
(iv) The target *replacement ratio*, r, is 65%.
(v) At age 65, the employee is still *married* and his wife's age is 61.
(vi) The proceeds (or accumulated contributions) at retirement will be used to purchase
(1) a *pension for lifetime* and (2) a *reversionary annuity* for the employee's wife at 60% of his pension.
(vii) Survival follows Makeham law with force of mortality $\mu_x = A + Bh^x$ where

	A	B	h
Male Survival	0.0004	4.05×10^{-6}	1.13
Female Survival	0.0002	10^{-6}	1.135

(viii) Members and their spouses are **independent** with respect to mortality.
(x) Monthly annuities purchased at retirement are priced at an interest rate of 5.5% per year, giving

$$\ddot{a}_{\overset{m}{65}} = 10.986\,599,\ \ddot{a}^{(12)}_{\underset{65:61}{m\ f}} = 10.0066, \text{ and } \ddot{a}^{(12)}_{\underset{61}{f}} = 13.9194.$$

Consider a new male entrant whose age at hire is **exactly** 25 and with beginning annual salary $S = 180000$.

a) Estimate the employee's **annual salary** when his age is 50.
b) Calculate the **contribution rate** c required to meet the target replacement ratio for this member.

4. (4+5=9 points) An employer establishes a *Defined Benefit* (DB) pension plan. The *normal retirement age* (*NRA*) is mandatory at 65, so $q_{65}^{(r)} = 1$. The retirement benefit is 1% of final *three-year average* salary per year of service. The following assumptions are made:

   (i) The employee is eligible for *early retirement* at age 61, with a 3% *reduction* per year early.

   (ii) The *withdrawal* benefit is the then accrued benefit otherwise payable at *NRA*. Vesting occurs after five years of service and no benefits will be paid before that time. On withdrawal from the plan for any reason after vesting and before normal retirement age of 65, the accrued benefits are paid to the employee.

   (iii) The annuity function under an interest rate of 0.06 is $\ddot{a}_{65} = 11.169\,22$.

   (iv) The death benefit requires *ten years of service*, and is set at 50% of the then accrued benefit, reduced as for early retirement.

   (v) Assume early retirement or withdrawal occur half way through the year of age, on average.

   Consider a newly hired employee age 51. The salary at hire is $100,000$ and is projected to increase by 4% per year.

(a) Find the *Projected Annual Benefit* at the normal retirement age (*NRA*) of 65.
(b) Using an interest rate of 6% per year, calculate the price at *age* 65 of the monthly annuity for normal retirement.

5. (5+3+2=10 points) A universal life insurance contract with fixed death benefit *face amount* of $100,000$ has an *account value* of 4000 on April 30. A *contribution* of 1000 is made on May 1. The annual *credited interest* rate is 4.5%, the percent of contribution expense rate is 4.0%, the monthly administrative *expense charge* is 40, and the *monthly mortality* rate is 0.0001. The *surrender charge* at this duration of the contract is 10 per 1000 of face amount. There is *no* outstanding *loan balance*. Calculate, as of May 31, each of

   (a) the account value, AV_t,

   (b) the cash value, CV_t, and

   (c) the cash surrender value, CSV_t.

6. (10 points) Consider a UL contract with **secondary guarantee** provided by the *shadow fund* method. As of time t the *shadow fund balance* is $60,000$, the *net single premium* required to fully fund the guarantee is $100,000$, the *valuation net single premium* is $150,000$, the applicable *surrender charge* is 4000, and the *basic reserve* is $10,000$. The *deficiency reserve* is 7500. Calculate the *AG38 reserve* for this contract.

7. (6+4+3+3=16 points) For a fully discrete 5-year term insurance of $1,000,000$ face amount, gross premium is $19,250$ with pre-contract expenses of 5000, annual per policy expense of 240 (payable at the beginning of the year), an interest rate of $i = 0.06$ on invested assets, and a risk discount rate of $r = 0.10$. There are no surrenders. The table below gives values of the terminal reserves and the mortality factors for this contract.

Policy year t	q_{x+t-1}	p_{x+t-1}	${}_tp_x$	${}_tV^G$
1	0.015	0.985	0.985	2500
2	0.017	0.983	0.96826	4000
3	0.019	0.981	0.94986	5000
4	0.021	0.979	0.92991	4000
5	0.024	0.976	0.90759	0

Calculate the following:
a) The Profit vector,
b) The Profit signature,
c) The Net Present Value (NPV), and
d) The Profit Margin.

8. (1+4=5 points) For a universal life policy, you are given:

(i) The **account value** on 1 January 2012 is \$100,000.
(ii) On January 1 2012, the policyholder pays a *premium* of \$3000. *No more premium* is paid in 2012.
(iii) The *expense charge* is \$100 + 2% of the premium.
(iv) The cost of insurance (or *policy charge*) deducted from the premium is \$980.
(v) The *credited interest rate* for year 2012 is 5% **per annum effective**.
(vi) The policy is still in force on 31 December 2012.
Calculate the **account value** on 31 December 2012.

A) \$106,953
B) \$107,128
C) \$108,850
D) \$110,117
E) \$111,508

Work Shown (4 points):

Hence the answer is (___).

9. (1+4=5 points) You are given:

(i) (30) and (50) have independent future lifetimes, subject to a 0.05 force of mortality each.
(ii) $\delta = 0.03$.
Calculate. $\overline{A}^{\,1}_{30:50}$.

(A) 0.23
(B) 0.38
(C) 0.51
(D) 0.64
(E) 0.77

Work Shown (4 points):

Hence the answer is (___)

END OF TEST PAPER