Dept of Mathematics and Statistics
King Fahd University of Petroleum & Minerals

AS482: Actuarial Contingencies II

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Major 1 Exam   Term 161   FORM A
Wednesday Oct 24 2016
6.00pm-7.20pm

Name______________________________ ID#:__________ Serial #:___

Instructions.

1. Please turn off your cell phones and place them under your chair. Any student caught with mobile phones on during the exam will be considered under the **cheating rules** of the University.
2. If you need to leave the room, please do so quietly so not to disturb others taking the test. No two person can leave the room at the same time. No extra time will be provided for the time missed outside the classroom.
3. Only materials provided by the instructor can be present on the table during the exam.
4. Do not spend too much time on any one question. If a question seems too difficult, leave it and go on.
5. Use the blank portions of each page for your work. Extra blank pages can be provided if necessary. If you use an extra page, indicate clearly what problem you are working on.
6. Only answers supported by work will be considered. Unsupported guesses will not be graded.
7. While every attempt is made to avoid defective questions, sometimes they do occur. In the rare event that you believe a question is defective, the instructor cannot give you any guidance beyond these instructions.
8. Mobile calculators, I-pad, or communicable devices are disallowed. Use regular scientific calculators or financail calculators only. Write important steps to arrive at the solution of the following problems.

The test is 80 minutes, GOOD LUCK, and you may begin now!

Question	Total Marks	Marks Obtained	Comments
1	4+2=6		
2	4+2=6		
3	6		
4	3+3=6		
5	3+7=10		
6	1+5=6		
Total	40		

Extra blank page

1. (4+2=6 points) A decrement model consist of only two states with with constant force of transition function $\lambda_{12}(s) = \lambda_1(s) = \lambda$ and $\lambda_{21}(s) = 0$.

   a) **Solve** Kolmogorov's differential equation for ${}_np_{12}^{(t)}$ for a 2-state homogeneous Markov survival model.

   b) Provide the **meaning** of ${}_np_{12}^{(t)}$ and then translate this into **standard actuarial notation**.

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2. (4+2=6 points) Consider a simple two-state model with transition probabilities given by $P^{(0)} = \begin{bmatrix} 0.60 & 0.40 \\ 0.70 & 0.30 \end{bmatrix}$ over the first interval of the process, and $P^{(1)} = \begin{bmatrix} 0.50 & 0.50 \\ 0.80 & 0.20 \end{bmatrix}$ for the second interval of the process.

a) If the process is known to begin in **State 1** at time 0, what is the probability that the process will be in **State 2** at **time 2**?
b) What **type of stochastic process** model best describes this model?

3. (6 points) The career of a 50 year old Professor of Actuarial Science is subject to two decrements. **Decrement 1** is **mortality**, which is governed by a **uniform survival** distribution with $\omega = 100$, and **Decrement 2** is **leaving** academic employment, which is governed by the **force of failure** (HRF) $\mu_y^{(2)} = 0.05$, for all $y \geq 50$.

   Find the probability that this professor remains in academic employment for **at least four years** but **less than ten** years.

4. (3+3=6 points) Graduate Students can leave a certain three-year school only for reasons *of failure* (Decrement 1) or *voluntary withdrawal* (Decrement 2), where each decrement is uniformly distributed over $(x, x+1)$ in its **associated single-decrement** table. The following values are given

x	$q_x^{\prime(1)}$	$q_x^{\prime(2)}$	$q_x^{(1)}$	$q_x^{(2)}$
0	.1	.25		
1	.2	.2		
2	.25	.1		

(a) Given that a person **decrements** from school in the **third** year, find the probability that the decrement was a **voluntary withdrawal**.

(b) Given that a student **enters Year 2**, find the probability of eventually decrementing due **to failure**.

5. (3+7=10 points) .A person is currently **employed** at time 0, which we call **State 1**. Let State **2** denote **unemployment** and State **3** denote **deceased**. The transition forces between states are as follows:-

(i) $\lambda_{12}(s) = 0.20 + 0.0002s^2$
(ii) $\lambda_{13}(s) = \lambda_{23}(s) = 0.05$
(iii) $\lambda_{21}(s) = 0.80 - 0.04s$
(iv) $\lambda_{31}(s) = \lambda_{32}(s) = 0$

(a) Draw the **transition state diagram** for this model.

(b) Using half-year time steps to approximate the solutions to the Kolmogorov differential equation, estimate${}_rp_{11}^{(0)}$ and ${}_rp_{12}^{(0)}$ for $r = 0.5, 1.0, .., 10.0$ by completing blank cells in the following table. (Be sure to provide at least one example calculations for every type of estimated probability)

r	${}_rp_{11}^{(0)}$	${}_rp_{12}^{(0)}$
0.5	0.875	0.100
1.0	0.805	0.146
1.5		0.167
2.0		0.177
2.5	0.699	0.182
3.0	0.675	
3.5	0.653	0.185
4.0	0.631	0.185
4.5	0.611	0.186
5.0	0.591	0.186

r	${}_rp_{11}^{(0)}$	${}_rp_{12}^{(0)}$
5.5	0.571	0.186
6.0	0.552	0.186
6.5	0.533	0.187
7.0		0.187
7.5	0.496	
8.0	0.478	
8.5	0.461	0.189
9.0	0.443	0.191
9.5		0.192
10.0	0.409	0.193

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6. (1+5=6 points) You are the pricing actuary reviewing cash values **(or withdrawal benefit)** on fully discrete whole life insurances of 10000 on (40). A desired asset share pattern has been chosen. You are to determine cash values that will produce those asset shares. You are given:

(i) The gross or contract premium is 90.
(ii) Renewal expenses, payable at the start of the year, are 5% of the premium.
(iii) $q_{55}^{(death)} = 0.004$
(iv) $q_{55}^{(withdrawal)} = 0.050$
(v) $i = 0.08$.
(v) ${}_{15}AS = 1150$ and ${}_{16}AS = 1320$ are the asset shares at the end of years 15 and 16.

Calculate ${}_{16}CV = b_{16}^{(withdrawal)}$, the **cash value (or benefit)** upon withdrawal at the end of year 16.

a) 810
b) 860
c) 910
d) 960
e) 1010

Work Shown (5 points):

Hence the answer is ( )

END OF TEST PAPER