Dept of Mathematics and Statistics
King Fahd University of Petroleum & Minerals

AS381: Actuarial Contingencies I

Dr. Mohammad H. Omar
Final Exam   Term 161   FORM A
Thursday January 19 2017
8.00am-10.30am

Name____ ID#:__________ Serial #:___

Instructions.

1. Please turn off your cell phones and place them under your chair. Any student caught with mobile phones on during the exam will be considered under the **cheating rules** of the University.
2. If you need to leave the room, please do so quietly so not to disturb others taking the test. No two person can leave the room at the same time. No extra time will be provided for the time missed outside the classroom.
3. Only materials provided by the instructor can be present on the table during the exam.
4. Do not spend too much time on any one question. If a question seems too difficult, leave it and go on.
5. Use the blank portions of each page for your work. Extra blank pages can be provided if necessary. If you use an extra page, indicate clearly what problem you are working on.
6. Only answers supported by work will be considered. Unsupported guesses will not be graded.
7. While every attempt is made to avoid defective questions, sometimes they do occur. In the rare event that you believe a question is defective, the instructor cannot give you any guidance beyond these instructions.
8. Mobile calculators, I-pad, or communicable devices are disallowed. Use regular scientific calculators or financail calculators only. Write important steps to arrive at the solution of the following problems.

The test is 150 minutes, GOOD LUCK, and you may begin now!

Question	Total Marks	Marks Obtained	Comments
1	3+3+4=10		
2	6		
3	4+4=8		
4	3+4+4=11		
5	3+12×0.5+2=11		
6	4+4=8		
7	3+2+3+3=11		
8	4+1=5		
9	4+1=5		
Total	75		

Extra blank page

1. (3+3+4=10 points) If ${}_{k|}q_x = c(0.94)^{k+1} \qquad k = 0, 1, 2, ...$

where $c = 0.06/0.94$ and $i = 0.05$,
calculate
a) A_x
b) P_x and
c) $Var(L)$.

2. (3+3=6 points) A fully continuous whole life insurance with unit benefit has a level premium. The time-until-death random variable, $T(x)$ has an exponential distribution with $E[T(x)] = 50$ and the force of interest is $\delta = 0.06$.

(a) If the principle of equivalence is used, find the benefit **premium** rate.
(b) Find the **premium** rate if it is to be such that $\Pr(L > 0) = 0.50$.

3. (4+4=8 points) An insurer is planning to issue a policy to a life age 0 whose curtate future lifetime K is governed by the p.f

$$_{k|}q_0 = 0.2 \qquad k = 0, 1, 2, 3, 4.$$

The policy will pay 1 unit at the end of the year of death in exchange for the payment of a premium P at the beginning of the year, provided the life survives. The insurer uses an annual effective interest rate $i = 0.05$.

a) Find the annual premium P such that the insurer, using a utility of wealth function $u(x) = x$, will be indifferent between accepting and not accepting the risk.

b) Determine the benefit reserve for $t = 2$ and 5 for this insurance policy.

4. (3+4+4=11 points) On the basis of the Illustrative Life Table, uniform distribution of death (UDD), and interest of 6%, calculate values for the benefit reserves in the following table.

a) **Fully Discrete**	b) **Semicontinuous**	c) **Fully Continuous**
${}_{10}V_{35}$	${}_{10}V\left(\bar{A}_{35:\overline{30}\vert}\right)$	${}_{10}\bar{V}\left(\bar{A}_{35}\right)$

5. ($3+12\times 0.5+2=11$ marks) A 4 year term life insurance of 5000 is issued on a fully discrete basis to each member of a group of l_{36} persons at age 36.

a) Trace the cash-flow expected for this group on the basis of the illustrative Life Table with interest at 6% and, as a by-product, obtain the benefit reserves.

To help you, part of the work is already shown below. **Complete** the work by filling in the blanks with the correct numbers. (show an example calculation work for each column)

Yr h	Expected Benefit Premiums at Start of Year	Expected Fund at **Start** of Year	Expected Interest	Expected Death Claims	Expected Fund at Year **End**	Expected Number of Survivors at Year **End**	$5000\times {}_hV^{\,1}_{36:\overline{4}\rvert}$
1	1042476.49		62548.59		98935.58		
2		1139180.90	68350.85			93601.83	1.4792
3		1176331.82		1137887.50	109024.23	93374.25	
4	1035351.11	1144375.34	68662.52	1213037.86		93131.64	
example calculation							

(b) Assume instead the contract is a 4 year **endowment** insurance of 5000 issued on a fully discrete basis to each member of a group of l_{36} persons at age 36. What should be the reserve at the end of year 4?

6. (4+4=8 points) For a five-year term insurance with benefit of 1000, the following are known:

(i) the premium is 6.5569128 and the interest rate is $i = 0.06$,

(ii) the reserves are ${}_1V^{1}_{x:\overline{5}|} = 1.0366$, ${}_2V^{1}_{x:\overline{5}|} = 1.6375$, ${}_3V^{1}_{x:\overline{5}|} = 1.7257$, ${}_4V^{1}_{x:\overline{5}|} = 1.2132$,

(iii) mortality are given in the table below

k	0	1	2	3	4	5
l_{x+k}	89509	88979.11	88407.68	87791.26	87126.2	86408.6
d_{x+k}	529.8844	571.4316	616.4165	665.0646	717.6041	774.2626

Consider an insured who has survived to the end of the second policy year. For this insured, evaluate

a. $Var\left[{}_3L|K(x) \geq 3\right]$ directly

b. $Var\left[{}_3L|K(x) \geq 3\right]$ by means of the Hattendorf theorem.

7. (3+2+3+3=11 points) A 10 payment whole life policy with unit benefit (or face) amount was issued on a fully discrete basis to a person age 25. On the basis of your Illustrative Life Table and interest of 6%, calculate

(a) ${}_{10}P_{25}$ (b) ${}^{10}_{9}V_{25}$ (c) $Var[{}_{10}L|K(25) \geq 10]$

(d) Given that ${}^{10}_{10}V_{25} = A_{35}$, find $Var[{}_{8}L|K(25) \geq 8]$ using the **Hattendorf's** theorem.

8. (4+1=5 points) For a special fully discrete 20-year endowment insurance on (40):

(i) The death benefit is 1000 for the first 10 years and 2000 thereafter. The pure endowment benefit is 2000.

(ii) Determined by the equivalence principle, the annual benefit premium is 40 for each of the first 10 years and 100 for each year thereafter.

(iii) $q_{40+k} = 0.001k + 0.001$, $k = 8, 9, \cdots, 13$

(iv) $i = 0.05$

(v) $\ddot{a}_{51:\overline{9}|} = 7.1$.

Calculate the 10th year terminal reserve using the benefit premiums.

a) 490

b) 500

c) 530

d) 550

e) 560

Work Shown (4 points)

. Hence the answer is (__)

9. (4+1=5 points) You are given:

(i) $\mu_x = F + e^{2x}, \quad x \geq 0$

(ii) ${}_{0.4}p_0 = 0.50$.

Calculate F.

a) -0.20 b) -0.09 c) 0.00 d) 0.09 e) 0.20

Work Shown (4 points)

Hence the answer is (__)

END OF TEST PAPER

9. (3+3+4=10 points) A fully continuous whole life insurance with unit benefit has a level premium. The time-until-death random variable, $T(x)$ has an exponential distribution with $E[T(x)] = 50$ and the force of interest is $\delta = 0.06$.

(a) If the principle of equivalence is used, find the benefit **premium** rate.
(b) Find the **premium** rate if it is to be such that $\Pr(L > 0) = 0.50$.
(c) Repeat part (b) if the force of interest, δ equals 0.

Solution: *see Bowers Chapter 6 Exercise 6.4 pg 197.*

(a) $\bar{A}_x = \int_0^\infty v^t\, {}_tp_x\, \mu_x(t)dt = \int_0^\infty e^{-\delta t} e^{-\mu t} \mu dt = \dfrac{\mu}{\mu+\delta} \int_0^\infty (\mu+\delta)e^{-(\mu+\delta)t}dt = \dfrac{\mu}{\mu+\delta}$.

$$\overline{P}(\bar{A}_x) = \frac{\delta \bar{A}_x}{1-\bar{A}_x} = \frac{\delta\left(\dfrac{\mu}{\mu+\delta}\right)}{1-\left(\dfrac{\mu}{\mu+\delta}\right)} = \frac{\delta\left(\dfrac{\mu}{\mu+\delta}\right)}{\left(\dfrac{\delta}{\mu+\delta}\right)} = \mu = 0.02 \text{ since } 50 = E[T] = \frac{1}{\mu}.$$

(b) $L = e^{-\delta T} - P\overline{a}_{\overline{T}|}$ is a decreasing function of T. So the 50th percentile of $L, 0$ is L evaluated at the 50th percentile of T as shown here $P\left(L > 0\right) = 0.5 \Leftrightarrow P\left(e^{-\delta T} - P\overline{a}_{\overline{T}|} > 0\right) = 0.5$

$$\begin{aligned}
P\left(e^{-\delta T}\left(1+\frac{P}{\delta}\right) - \frac{P}{\delta} > 0\right) &= 0.5 \\
P\left(e^{-\delta T} > \frac{P}{\delta}\Big/\left(1+\frac{P}{\delta}\right)\right) &= 0.5 \\
P\left(T < -\frac{1}{\delta}\ln\left(\frac{P}{\delta}\Big/\left(1+\frac{P}{\delta}\right)\right)\right) &= 0.5 \\
1 - e^{-\mu t_0} &= 0.5 \quad \text{where } t_0 = -\frac{1}{\delta}\ln\left(\frac{P}{\delta}\Big/\left(1+\frac{P}{\delta}\right)\right) \\
t_0 &= \frac{-1}{\mu}\ln(1-0.5) = \frac{1}{\mu}\ln 2.
\end{aligned}$$

So, $t_0 = \dfrac{\ln 2}{\mu} = \dfrac{\ln 2}{0.02} = 34.657359 \approx 34.66$ years and substituting into L:

$$0 = e^{-0.06(34.66)} - P\overline{a}_{\overline{25}|} \Longrightarrow P = \frac{e^{-0.06(34.66)}}{\overline{a}_{\overline{25}|}} = \frac{1}{\overline{s}_{\overline{25}|}} = 0.0086.$$

(c) $0 = 1 - P\,\overset{o}{e}_x \Longrightarrow P = \dfrac{1}{\overset{o}{e}_x} = \mu = 0.02$ since $e^{\delta t} = 1$ and

$$\begin{aligned}
E\left[\overline{a}_{\overline{T}|}\right] &= \int_0^\infty v^t\, {}_tp_x dt = \int_0^\infty e^{-\delta t}e^{-\mu t}dt = \int_0^\infty e^{-\mu t}dt \text{ since } e^{\delta t} = 1 \\
&= \int_0^\infty \left(1 - F_T(t)\right)dt = E[T] = \overset{o}{e}_x.
\end{aligned}$$