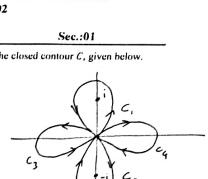
King Fahd University of Petroleum and Minerals
MATH-302
Quiz 6

Name:- **ID:-** **Sec.:01**

(1) Evaluate the integral $\oint_C \frac{e^{iz}}{(z^2+1)^2}\,dz$ along the the closed contour C, given below.



C_1: $\displaystyle\oint_{C_1} \frac{\frac{e^{iz}}{(z+i)^2}}{(z-i)^2}\,dz$

$$= \frac{2\pi i}{1!}\left(\frac{e^{iz}}{(z+i)^2}\right)'_{z=i}$$

$$= 2\pi i\left(\frac{ie^{iz}(z+i)^2 - e^{iz}\,2(z+i)}{(z+i)^4}\right)_{z=i} = 2\pi i\,\frac{\left(ie^{-1}\cdot(-4) - e^{-1}\cdot 4i\right)}{2^4}$$

$$= \pi e^{-1}$$

$\displaystyle\oint_{C_2} \frac{\frac{e^{iz}}{(z-i)^2}}{(z+i)^2}\,dz = -2\pi i\left(\frac{e^{iz}}{(z-i)^2}\right)'_{z=-i}$ negative orientation

$$= -2\pi i\left(\frac{ie^{iz}(z-i)^2 - 2e^{iz}(z-i)}{(z-i)^4}\right)_{z=-i} = -2\pi i\,\frac{\left(ie^{-1}(-4) - 2e^{-1}(-2i)\right)}{2^4}$$

$$= 0$$

$= \oint_{C_4} = 0$

$$\oint_C = \int_{C_1} + \int_{C_2} + \int_{C_3} + \int_{C_4} = \pi e^{-1}$$

(2) Find all values of z satisfying $\sinh z = -1$.

$$\frac{e^z - e^{-z}}{2} = -1$$

$$e^{2z} + 2e^z - 1 = 0$$

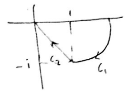
$$e^z = \frac{-2 \mp \sqrt{4+4}}{2} = -1 \mp \sqrt{2}$$

$$z = \ln(-1 \mp \sqrt{2}) = \log_e|-1+\sqrt{2}| + i\,2\pi n$$

$$= \log_e(\sqrt{2}-1) + i\,2\pi n$$

$$z = \ln(-1-\sqrt{2}) = \log_e(1+\sqrt{2}) + i(\pi + 2\pi n)$$

(3)



C_1: $z = 1 + e^{i\theta}$ $\quad \frac{3\pi}{2} \le \theta \le 2\pi$

$$-\int_{3\pi/2}^{2\pi} e^{i\theta}(ie^{i\theta})\,d\theta = -i\,\frac{e^{2i\theta}}{2i}\Bigg|_{3\pi/2}^{2\pi} = -\frac{1}{2}\left(\overset{1}{e^{4\pi i}} - \overset{-1}{e^{3\pi i}}\right)$$

$$= -\frac{2}{2} = -1$$

C_2: $z = t - it$ $\quad 0 \le t \le 1$

$$-\int_0^1 (t - it - 1)(1 - i)\,dt$$

$$= \int_0^1 t + i(1-2t)\,dt = 1$$

$$\int_{C_1} + \int_{C_2} = 0$$