King Fahd University of Petroleum and Minerals
MATH-302
Quiz 3

Name:- **ID:-** **Sec.:01**

(1) Let $S = \{(x, y, z, t) | x + y = z,\ z = 5t\}$.

(a) Show that S is a subspace of R^4

(b) Find a basis and the dimension of S.

(a) General form of a vector is S is $\langle x, y, x+y, \frac{x+y}{5} \rangle$

(i) $\vec{a} = \langle a_1, a_2, a_1+a_2, \frac{a_1+a_2}{5} \rangle \in S$

$\vec{b} = \langle b_1, b_2, b_1+b_2, \frac{b_1+b_2}{5} \rangle \in S$

$\vec{a} + \vec{b} = \langle \underbrace{a_1+b_1}_{x}, \underbrace{a_2+b_2}_{y}, \underbrace{(a_1+b_1)}_{x} + \underbrace{(a_2+b_2)}_{y}, \frac{\overbrace{(a_1+b_1)}^{x} + \overbrace{(a_2+b_2)}^{y}}{5} \rangle \in S$

(ii) $k\vec{a} = \langle ka_1, ka_2, ka_1 + ka_2, k\frac{a_1+a_2}{5} \rangle \in S$

because $k\left(\frac{a_1+a_2}{5}\right) = \frac{ka_1+ka_2}{5}$.

(b) $\langle a_1, a_2, a_1+a_2, \frac{a_1+a_2}{5} \rangle = a_1\langle 1,0,1,\frac{1}{5}\rangle + a_2\langle 0,1,1,\frac{1}{5}\rangle$

$\langle 1,0,1,\frac{1}{5}\rangle$, $\langle 0,1,1,\frac{1}{5}\rangle$ are not multiple of each other.

So, they are lin. independent.

$\{\langle 1,0,1,\frac{1}{5}\rangle, \langle 0,1,1,\frac{1}{5}\rangle\}$ is a basis.

$\Rightarrow$ Dimension is 2.

(2) (a) Determine whether the given set of vectors linearly dependent or linearly independent.

(i) $\underset{a}{\langle 1,1,-3,0,4\rangle}, \underset{b}{\langle 2,-1,-4,-6,2\rangle}, \underset{c}{\langle 0,-3,2,-6,-6\rangle}$

(ii) $\langle 1,1,-3,0\rangle, \langle 2,-1,-4,-6\rangle, \langle 0,-3,2,-1\rangle$

(i) $\vec{b} - 2\vec{a} - \vec{c} = 0 \Rightarrow$ They are linearly dependent.

(ii) $\alpha_1\langle 1,1,-3,0\rangle + \alpha_2\langle 2,-1,-4,-6\rangle + \alpha_3\langle 0,-3,2,-1\rangle = 0$

$$\left.\begin{aligned} \alpha_1 + 2\alpha_2 &= 0 \\ \alpha_1 - \alpha_2 - 3\alpha_3 &= 0 \\ -3\alpha_1 - 4\alpha_2 + 2\alpha_3 &= 0 \\ -6\alpha_2 - \alpha_3 &= 0 \end{aligned}\right\} \Rightarrow \alpha_1 = \alpha_2 = \alpha_3 = 0$$

They are lin. independent.

(b) Use Gaussian Elimination method to solve the given system or show that no solution exists.

$$x_1 + 2x_2 - 4x_3 = 9$$
$$5x_1 - x_2 + 2x_3 = 1$$

$$\begin{pmatrix} 1 & 2 & -4 & 9 \\ 5 & -1 & 2 & 1 \end{pmatrix} \xrightarrow{R_2 - 5R_1} \begin{pmatrix} 1 & 2 & -4 & 9 \\ 0 & -11 & 22 & -44 \end{pmatrix}$$

$$\xrightarrow{-\frac{1}{11}R_2} \begin{pmatrix} 1 & 2 & -4 & 9 \\ 0 & 1 & -2 & 4 \end{pmatrix}$$

$\boxed{x_3 = t \quad x_2 = 2t + 4}$

$x_1 + 2(2t+4) - 4t = 9 \Rightarrow \boxed{x_1 = 1}$