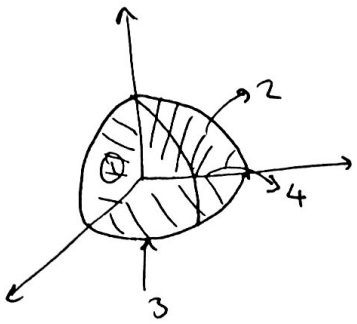
King Fahd University of Petroleum and Minerals
MATH-302
Quiz 2

Name:- **ID:-** **Sec.:01**

(1) Verify divergence theorem for $\mathbf{F} = r^2\boldsymbol{a}_r + r\sin\theta\cos\varphi\,\boldsymbol{a}_\theta$ over the surface defined by $0 < r < 3, 0 < \varphi < \pi/2, 0 < \theta < \pi/2$.

Hint.

$$\nabla \cdot \mathbf{G} = \frac{1}{r^2}\frac{\partial(r^2 G_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta\, G_\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial(G_\phi)}{\partial\phi}$$



There are 4 surfaces

① $\varphi = 0 \quad d\varphi = 0$

$dS = -r\,dr\,d\theta\,a_\varphi \quad \int F\cdot dS = \iint 0 = 0$

② $\varphi = \pi/2 \quad d\varphi = 0 \quad F\cdot dS = 0 \quad \int F\cdot dS = 0.$

③ $r = 3 \quad dr = 0 \quad dS = r^2\sin\theta\,d\theta\,d\varphi\,\bar{a}_r = 9\sin\theta\,d\theta\,d\varphi\,\bar{a}_r$

$$\int F\cdot dS = \int_0^{\pi/2}\int_0^{\pi/2} 81\sin\theta\,d\theta\,d\varphi = 81\pi/2$$

④ $\theta = \pi/2 \quad d\theta = 0 \quad dS = r\sin\theta\,dr\,d\varphi\,\bar{a}_\theta = r\,dr\,d\varphi\,\bar{a}_\theta$

$$\int F\cdot dS = \int\int r^2\cos\varphi\,dr\,d\varphi = 9$$

$$① + ② + ③ + ④ = 81\pi/2 + 9$$

$$\nabla\cdot F = 4r + 2\cos\theta\cos\varphi$$

$$\int \nabla\cdot F\,dv = \int_0^{\pi/2}\int_0^{\pi/2}\int_0^{3} (4r + 2\cos\theta\cos\varphi)\, r^2\sin\theta\,dr\,d\theta\,d\varphi = 81\pi/2 + 9.$$