King Fahd University of Petroleum and Minerals

MATH-302

Quiz 1

Name:- **ID:-** **Sec.:01**

(1) Express vector $\mathbf{G} = \underbrace{\rho \sin\varphi}_{A_\rho}\, \boldsymbol{a}_\rho - \underbrace{\rho\cos\varphi}_{A_\varphi}\, \boldsymbol{a}_\varphi + \underbrace{\rho}_{A_z} \boldsymbol{a}_z$ in rectangular coordinates.

$$\begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} = \begin{pmatrix} \cos\varphi & -\sin\varphi & 0 \\ \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} A_\rho \\ A_\varphi \\ A_z \end{pmatrix}$$

$$A_x = \rho\sin\varphi\cos\varphi + \rho\sin\varphi\cos\varphi = 2\rho\sin\varphi\cos\varphi = \frac{2xy}{\sqrt{x^2+y^2}}$$

$$A_y = \rho\sin^2\varphi - \rho\cos^2\varphi = y\sin\varphi - x\cos\varphi$$

$$= \frac{y^2-x^2}{\sqrt{x^2+y^2}}$$

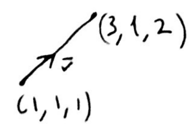
$$A_z = \rho = \sqrt{x^2+y^2}$$

$$G = \frac{2xy}{\sqrt{x^2+y^2}}\,\bar{a}_x + \frac{y^2-x^2}{\sqrt{x^2+y^2}}\,\bar{a}_y + \sqrt{x^2+y^2}\,\bar{a}_z$$

(2) In a force field, $\mathbf{F} = (x-y)\boldsymbol{a}_x - (x^2+zy)\boldsymbol{a}_y + 2yz\boldsymbol{a}_z$ find the work done in moving a particle along the line from (1,1,1) to (3,1,2).

$$d\ell = dx\,\bar{a}_x + dy\,\bar{a}_y + dz\,\bar{a}_z$$

L:



$$\vec{v} = (3,1,2) - (1,1,1)$$

$$\vec{v} = (2,0,1)$$

$$\boxed{\begin{aligned} L:\quad x &= 2t+1 = 2t+1 \\ y &= 0\cdot t+1 = 1 \qquad 0<t<1 \\ z &= t+1 = t+1 \end{aligned}}$$

$$\Rightarrow dx = 2dt \quad dy = 0 \quad dz = dt$$

$$W = \int_L F\cdot d\ell = \int_0^1 \underbrace{2t}_{(x-y)}\cdot 2dt + \cancel{*}\cdot 0 + \underbrace{2(t+1)}_{2yz}\,dt$$

$$= \int_0^1 (6t+2)\,dt = 3t^2+2t\Big|_0^1 = 5.$$