King Fahd University of Petroleum & Minerals
Department of Mathematics and Statistics

MATH 302, Semester 153 (2015-2016)

EXAM II
Wednesday, August 10, 2016

Allowed Time: 2 Hours

Student Name:

Student ID Number:

Section Number: 01

Instructor's Name: A. N. Duman

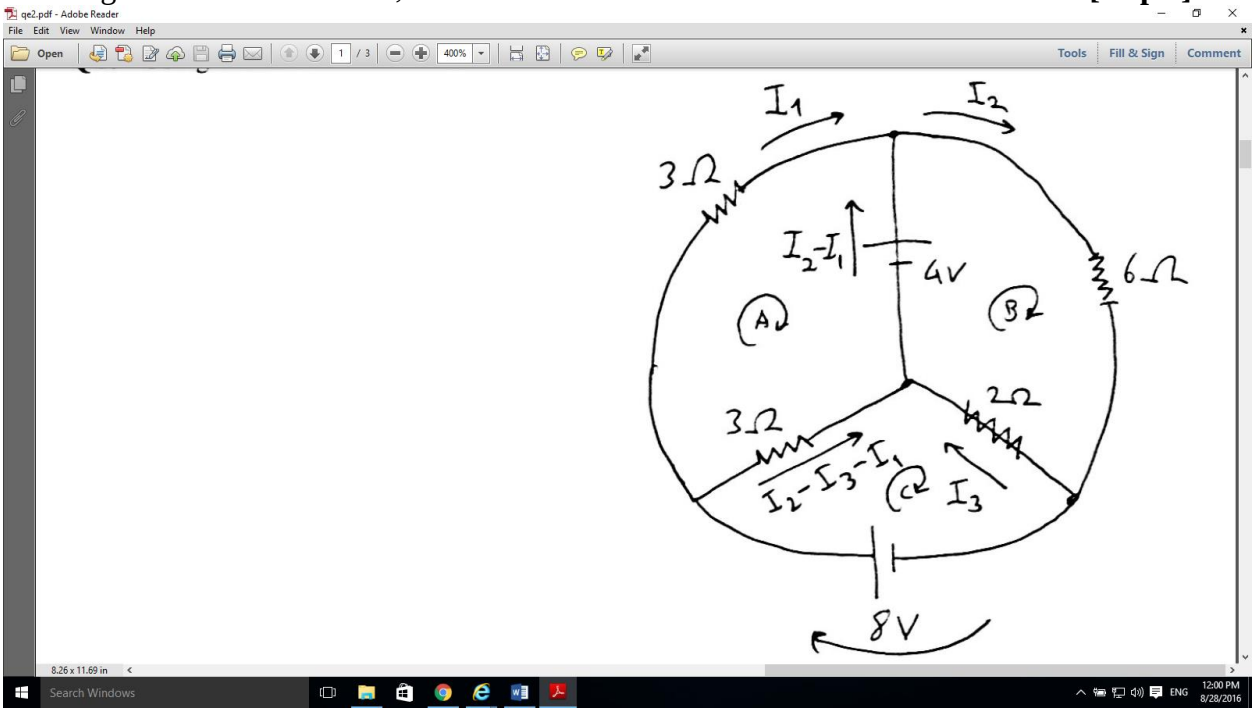
Instructions:
1. Write neatly and legibly -- *you may lose points for messy work.*
2. Show all your work -- *no points for answers without justification*.
3. Programmable Calculators and Mobiles are not allowed.
4. Make sure that you have 5 different problems (5 pages + cover page).

Problem No.	Points	Maximum Points
1		20
2		20
3		25
4		15
5		20
Total:		100

Q1. (a) Determine whether, $S = \{\langle x, y, z\rangle \mid x^2yz \geq 0\}$ is a subspace of $\mathbb{R}^3$? ***[5 pts]***

(b) Find a basis and for the subspace, $S = \{\vec{v} \in \mathbb{R}^4 \mid \vec{a} \cdot \vec{v} = 0 \text{ and } \vec{b} \cdot \vec{v} = 0\}$ of $\mathbb{R}^4$ where $\vec{a} = \langle 1,0,1,1\rangle$ and $\vec{b} = \langle 0,0,1,2\rangle$. What is the dimension of this subspace? ***[15 pts]***

Q2. Using Gaussian elimination, find the currents in all branches of the circuit below. **[20 pts]**



Q3. (a) Consider the system of non-homogenous linear algebraic equations,

$$\begin{aligned} x_1 &+ x_2 &+ x_3 &= r \\ ax_1 &+ a^2x_2 &+ a^2x_3 &= s \\ &x_2 &+ 2x_3 &= t \end{aligned}$$

If the system is inconsistent, what are the possible values of a? **[15 pts]**

(b) Let A be 4×6 matrix. If the solution of the system $AX = B$ has 2 parameters, what is **rank**(A) and **rank**(A| B)? **[5 pts]**

(c) Determine whether $A = \begin{pmatrix} 1 \\ 1 \\ 0 \\ -1 \end{pmatrix}$, $B = \begin{pmatrix} 1 \\ -1 \\ 1 \\ 0 \end{pmatrix}$, $C = \begin{pmatrix} 1 \\ 0 \\ 3 \\ 2 \end{pmatrix}$ are linearly independent.

[5 pts]

Q4. (a) Find $(AB)^{-1}$, if

$$A = \begin{pmatrix} 2 & -1 & 1 \\ 3 & 0 & -1 \\ 1 & 2 & -1 \end{pmatrix}, B^{-1} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

[10 points]

(b) Solve the system $(AB)X = C$,, where $X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$, $C = \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}$.

[5 points]

Q5. (a) Find the orthogonal matrix P that diagonalises the matrix A, where

$$A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$$

[10 points]

(b) Find P^{-1} ***[5 points]***

(c) Find $P^T AP$? ***[5 points]***