King Fahd University of Petroleum & Minerals
Department of Mathematics and Statistics

MATH 302, Semester 153 (2015-2016)

EXAM I

Wednesday, July 27, 2016

Allowed Time: 3 Hours

Student Name:

Student ID Number:

Section Number: 01

Instructor's Name: A. N. Duman

Instructions:

1. Write neatly and legibly -- *you may lose points for messy work*.
2. Show all your work -- *no points for answers without justification*.
3. Calculators and Mobiles are not allowed.
4. Make sure that you have 7 questions (8 pages + cover page + formula sheet).

Problem No.	Points	Maximum Points
1		20
2		10
3		15
4		15
5		20
6		10
7		10
Total:		**100**

Coordinator: Dr A. N. Duman

Q1. If $\boldsymbol{E} = 6r^2 \sin\theta \cos\varphi \,\widehat{\boldsymbol{a}}_{\boldsymbol{r}} + 4r \cos 2\theta \sin(\varphi/2)\, \widehat{\boldsymbol{a}}_{\boldsymbol{\theta}} + r^3 \widehat{\boldsymbol{a}}_{\boldsymbol{\varphi}}$ at $A(2, \frac{\pi}{6}, \frac{\pi}{3})$ determine the vector component of $\boldsymbol{E}$ that is:

(a) Tangential to the spherical surface $r = 2$. ***[5 pts]***

(b) Normal to the surface $\varphi = \pi/3$. ***[5 pts]***

(c) Parallel to the line $y = 2, z = 0$. ***[5 pts]***

(d) Determine a vector perpendicular to $\boldsymbol{E}$ and tangential to the plane $z = 2$. ***[5 pts]***

Q2.

(a) Find the magnitude and the direction of the maximum rate of change of $T = \frac{2}{r}\sin\theta\cos\varphi$ at the point $P(1, \frac{\pi}{6}, \frac{\pi}{2})$. **[5 pts]**

(b) Find $\nabla^2 V$ where is $V = \rho^2 z \sin 3\varphi$ in cylindrical coordinates where $\rho \neq 0$. **[5 pts]**

Q3. Express $\boldsymbol{G} = \frac{x\cos\varphi}{\rho}\hat{\boldsymbol{a}}_x + \frac{2yz}{\rho^2}\hat{\boldsymbol{a}}_y + (1 - \frac{x^2}{\rho^2})\hat{\boldsymbol{a}}_z$ completely in spherical system. **[10 pts]**

Q4. If $\boldsymbol{E} = 2xyz\,\hat{\boldsymbol{a}}_{\boldsymbol{x}} + x^2z\,\hat{\boldsymbol{a}}_y + x^2y\,\hat{\boldsymbol{a}}_{\boldsymbol{z}}$**,** show that the line integral L between two arbitrary points A and B in a simply connected region containing the path,

$$L: \int_A^B \boldsymbol{E} \cdot d\boldsymbol{l}$$

is independent of the path. Use the potential function to evaluate L where, $A = (2,1,-1)$, and $B = (5,1,2)$.

[15 pts]

Q5. Verify the divergence theorem for the function $\boldsymbol{E} = \rho^2 \hat{\boldsymbol{a}}_{\boldsymbol{\rho}} + z \cos\varphi\, \hat{\boldsymbol{a}}_{\varphi} - \rho \sin\varphi\, \hat{\boldsymbol{a}}_z$, over cylinder defined by $1 < \rho < 3,\ 0 < z < 4.$ ***[20 points]***

Q6. Using the curl of $\boldsymbol{E} = r\ \sin\theta\ \hat{\boldsymbol{a}}_{\boldsymbol{r}} + \cos\phi\,\hat{\boldsymbol{a}}_{\theta} - r^2\cos\theta\,\hat{\boldsymbol{a}}_{\boldsymbol{\varphi}}$ **,** evaluate the circulation,

$$\int_C \boldsymbol{E}\cdot d\boldsymbol{l}$$

around the closed path C which is the boundary of the open surface defined by $0 < \theta < \pi/2,\ 0 < \phi < \frac{\pi}{4},\ r = 2.$ ***[15 points]***

Q7. Let $\boldsymbol{E} = 2r\sin\theta\,\mathbf{cos}\,\boldsymbol{\varphi}\ \ \widehat{\boldsymbol{a}}_{\boldsymbol{r}} + r\cos\theta\cos\phi\,\widehat{\boldsymbol{a}}_{\theta} - r\sin\phi\,\widehat{\boldsymbol{a}}_{\boldsymbol{\varphi}}$ be the electric field on a certain region of space. Given two points $A(1,0,0)$ and $B(4,\pi/6,\pi/3)$ inside this region, find the electric potential at A, i.e. $V(A)$, given that $V(B) = 5$. ***[10 points]***

Formulae in cylindrical and spherical coordinate systems

Differential of displacement

Cylindrical: $d\boldsymbol{l} = d\rho\, \widehat{\boldsymbol{a}_\rho} + \rho d\phi\, \widehat{\boldsymbol{a}_\phi} + dz\, \widehat{\boldsymbol{a}_z}$

Spherical: $d\boldsymbol{l} = dr\, \widehat{\boldsymbol{a}_r} + r d\theta\, \widehat{\boldsymbol{a}_\theta} + r\sin\theta\, d\phi\, \widehat{\boldsymbol{a}_\phi}$

Gradient of a scalar field, ∇ V

Cylindrical: $\nabla V = \frac{\partial V}{\partial \boldsymbol{\rho}}\, \widehat{\boldsymbol{a}_\rho} + \frac{1}{\boldsymbol{\rho}}\frac{\partial V}{\partial \boldsymbol{\phi}}\, \widehat{\boldsymbol{a}_\phi} + \frac{\partial V}{\partial \boldsymbol{z}}\, \widehat{\boldsymbol{a}_z}$

Spherical: $\nabla V = \frac{\partial V}{\partial \boldsymbol{r}}\, \widehat{\boldsymbol{a}_r} + \frac{1}{\boldsymbol{r}}\frac{\partial V}{\partial \boldsymbol{\theta}}\, \widehat{\boldsymbol{a}_\theta} + \frac{1}{\boldsymbol{r}\sin\theta}\frac{\partial V}{\partial \boldsymbol{\phi}}\, \widehat{\boldsymbol{a}_\phi}$

Divergence of a vector field, $\nabla \cdot \mathbf{G}$

Cylindrical: $\nabla \cdot \mathbf{G} = \frac{1}{\boldsymbol{\rho}}\frac{\partial(\boldsymbol{\rho} G_{\boldsymbol{\rho}})}{\partial \boldsymbol{\rho}} + \frac{1}{\boldsymbol{\rho}}\frac{\partial(G_{\boldsymbol{\phi}})}{\partial \boldsymbol{\phi}} + \frac{\partial(G_z)}{\partial \boldsymbol{z}}$

Spherical: $\nabla \cdot \mathbf{G} = \frac{1}{r^2}\frac{\partial(r^2 G_{\boldsymbol{r}})}{\partial \boldsymbol{r}} + \frac{1}{\boldsymbol{r}\sin\theta}\frac{\partial(\sin\theta\, G_\theta)}{\partial\theta} + \frac{1}{\boldsymbol{r}\sin\theta}\frac{\partial(G_{\boldsymbol{\phi}})}{\partial \boldsymbol{\phi}}$

Relationship between Cartesian, Cylindrical and Spherical Coordinates

$$\begin{pmatrix} A_\rho \\ A_\varphi \\ A_z \end{pmatrix} = \begin{pmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix}$$

$$\begin{pmatrix} A_r \\ A_\theta \\ A_\varphi \end{pmatrix} = \begin{pmatrix} \sin\theta\cos\varphi & \sin\theta\sin\varphi & \cos\theta \\ \cos\theta\cos\varphi & \cos\theta\sin\varphi & -\sin\theta \\ -\sin\varphi & \cos\varphi & 0 \end{pmatrix} \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix}$$