Name:-                          ID:-                    Sec.:-        S.N.:-

Q1)

(a) Find the eigenfunctions and the equation that defines the the eigenvalues of the singular Sturm-Liouville problem

$$x^2 y'' + x y' + (\alpha^2 x^2 - 4) y = 0, \quad y(5) = 0, \quad y \text{ is bounded on } [0,5].$$

(5 points)

(b) Put the differential equation in self-adjoint form.

(3 points)

(c) Give the orthogonality relation.

(2 points)

Q2) Expand the given function in a Fourier-Bessel series using Bessel functions of the same order as in the indicated boundary condition.

$$f(x) = x, 0 < x < 3$$

$$J_1'(3\alpha) = 0$$

(5 points)