

King Fahd University of Petroleum & Minerals

Department of Mathematics & Statistics

2015-2016 (Term 153)

Introduction to Differential Equations & Linear Algebra

(MATH 260)

Exam II

Student Name:

Section #:

ID #:

Serial #:

Instructions

1. Justify your answers. No credit is given for (correct) answers not supported by work.
2. Write clearly. Marks may be deducted for messy work.

Question	Marks	Out of
1		14
2		10
3		8
4		8
5		14
6		8
7		14
8		14
Total		90

1. [14pts] Use the cofactor matrix of A to find A^{-1} if $A = \begin{bmatrix} -1 & 1 & 1 \\ 1 & 0 & -2 \\ 2 & -1 & 0 \end{bmatrix}$.

2. [10pts] Let $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & j \end{vmatrix} = 4$. Use elementary operations to find the value of the determinant

$$\begin{vmatrix} 2a - b & 3c & b \\ 2d - e & 3f & e \\ 2g - h & 3j & h \end{vmatrix}$$

3. [8pts] Let S consist of all vectors (x, y, z, w) of \mathbb{R}^4 such that $xy + z + w = 0$. Is S a subspace of \mathbb{R}^4 ? Justify.

4. [8pts] Find all possible values of k for which the vectors $(1, 2, 4)$, $(1, 3, 9)$, $(1, k, k^2)$ of \mathbb{R}^3 are linearly independent.

5. [14pts] Given that $y_1 = \cos 2x$ is a solution of the DE

$$y^{(4)} - y^{(3)} + 2y'' - 4y' - 8y = 0,$$

find:

- (a) a second solution y_2 of the DE such that y_1 and y_2 are linearly independent.
- (b) the general solution of the DE.

6. [8pts] Are the functions $f(x) = x$, $g(x) = e^x - 1$, $h(x) = xe^x$ linearly independent on $(-\infty, \infty)$? Justify.

7. [14pts] Solve the IVP: $y''' - 3y'' + 4y' - 2y = 0$, $y(0) = 2$, $y'(0) = 3$, $y''(0) = 4$.

8. [14pts] Find the form of a particular solution of the DE:

$$(D - 1)^2 (D^2 - 9) y = e^{3x} + 2e^{-3x} + 3xe^x.$$