

1. Apply the eigenvalue method to solve the system $x'_1 = 4x_1 + x_2 + x_3$, $x'_2 = x_1 + 4x_2 + x_3$, $x'_3 = x_1 + x_2 + 4x_3$.

2. The vectors $X_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} e^{2t}$, $X_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} e^{-t}$, $X_3 = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} e^{-t}$ are solutions of the system $X' = AX$,

where $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$.

(a) Verify that the vectors X_1, X_2, X_3 are linearly independent and that X_1 is a solution of the system.

(b) Solve the IVP: $X' = AX$, $X(0) = \begin{bmatrix} 10 \\ 12 \\ -1 \end{bmatrix}$.