King Fahd University of Petroleum & Minerals

Department of Mathematics & Statistics 2015-2016 (Term 153) Introduction to Differential Equations & Linear Algebra (MATH 260) FINAL EXAM (CODE 1)

Student Name:

Section #:

<u>ID</u> #:

Serial #:

Instructions

- 1. No electronic device (such as calculator, mobile phone, smart watch) is allowed in this exam.
- 2. Justify your answers for the <u>first seven</u> questions; no credit is given for (correct) answers not supported by work.
- 3. For the last five questions, circle the correct answer.
- 4. Write clearly. Marks may be deducted for messy work.

Question	Marks	Out of
1		15
2		15
3		15
4		15
5		15
6		15
7		10
8		8
9		8
10		8
11		8
12		8
Total		140

1. Use variation of parameters to solve the equation: $x^2y'' - 2xy' + 2y = x \ln x$ (x > 0), given that it has complementary function $y_c = c_1 x + c_2 x^2$.

2. Solve the system $X' = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 2 \\ 1 & 1 & 1 \end{bmatrix} X.$

3. Solve the IVP:
$$X' = AX$$
, $X(0) = \begin{bmatrix} 2\\3\\5 \end{bmatrix}$, where $A = \begin{bmatrix} 2 & 4 & 4\\-1 & -2 & 0\\-1 & 0 & -2 \end{bmatrix}$.

4. Solve the system X' = AX, where $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 2 & 1 \\ 0 & -1 & 0 \end{bmatrix}$.

5. Solve the system X' = AX, where $A = \begin{bmatrix} -1 & -8 & 5 \\ 3 & 10 & -5 \\ 3 & 8 & -3 \end{bmatrix}$, given that the eigenvalues of A are 2,2,2.

6. (a) Find all values of k for which the matrix $\begin{bmatrix} k & -1 & 0 \\ -1 & 1 & 1 \\ -2 & 1 & -k \end{bmatrix}$ is not invertible.

(b) Let $A = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 1 \\ -2 & 1 & 0 \end{bmatrix}$. Use Cayley-Hamilton theorem to find the inverse of A and the inverse of A^2 , if they exist.

7. Let v_1, v_2, v_3 be linearly independent vectors in \mathbb{R}^n . Are the vectors:

 $w_1 = 4v_1 + 2v_3, w_2 = v_1 - 3v_2 + 2v_3, w_3 = 4v_2 - 2v_3$

linearly dependent or independent? Justify.

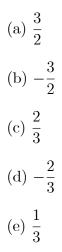
8. An object is placed in a 350 °F oven at 1:00 pm. At 1:01 pm the temperature of the object is 230 °F and at 1:02 pm its temperature is 260 °F. The initial temperature of the object is:

- (a) 170°F
- (b) 180°F
- (c) 190°F
- (d) $200 \,{}^{\circ}F$
- (e) 210°F

9. If y is a solution of the IVP: $x^{2}y' - xy = -y^{2}$, y(1) = 1, then y(e) =

- (a) 2*e*
- (b) *e*
- (c) $\frac{2e}{3}$ (d) $\frac{e}{2}$
- (e) $\frac{e}{3}$

10. If a 3×3 matrix A has eigenvalues 2, 3, and -6, then the sum of the eigenvalues of A^{-1} is:



11. If
$$A = \begin{bmatrix} 1 & 0 \\ -2 & 2 \end{bmatrix}$$
, then the sum of all the entries of A^{100} is:
(a) $3 - 2^{100}$
(b) $2^{100} - 3$
(c) $2^{100} - 1$
(d) $1 - 2^{100}$
(e) $2 - 2^{100}$

- 12. Which one of the following statements is TRUE for the given vectors?
 - (a) (1, -2, 0), (3, -2, 4), (5, -2, 8) form a basis of \mathbb{R}^3 .
 - (b) (1, 1, 1, 2), (2, 1, 1, 1), (1, 2, 1, 1) span \mathbb{R}^4 .
 - (c) (1, 1, 0, 1), (1, 0, 0, 1), (1, 1, 1, 1) are linearly dependent.
 - (d) (1, 0, 1, 0), (0, 1, 0, 1), (1, 0, 0, 1) form a basis of \mathbb{R}^4 .
 - (e) (1,0,1), (2,1,0), (3,1,1), (4,1,1) span \mathbb{R}^3 .