King Fahd University of Petroleum & Minerals

Department of Mathematics & Statistics
2015-2016 (Term 153)
Introduction to Differential Equations & Linear Algebra (MATH 260)
FINAL EXAM (CODE 1)

Student Name:	**Section #**:
ID #:	**Serial #**:

Instructions

1. **No electronic device (such as calculator, mobile phone, smart watch) is allowed in this exam.**
2. **Justify your answers for the first seven questions; no credit is given for (correct) answers not supported by work.**
3. **For the last five questions, circle the correct answer.**
4. **Write clearly. Marks may be deducted for messy work.**

Question	Marks	Out of
1		15
2		15
3		15
4		15
5		15
6		15
7		10
8		8
9		8
10		8
11		8
12		8
Total		140

1. Use variation of parameters to solve the equation: $x^2y'' - 2xy' + 2y = x \ln x$ $(x > 0)$, given that it has complementary function $y_c = c_1x + c_2x^2$.

2. Solve the system $X' = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 2 \\ 1 & 1 & 1 \end{bmatrix} X.$

3. Solve the IVP: $X' = AX$, $X(0) = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$, where $A = \begin{bmatrix} 2 & 4 & 4 \\ -1 & -2 & 0 \\ -1 & 0 & -2 \end{bmatrix}$.

4. Solve the system $X' = AX$, where $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 2 & 1 \\ 0 & -1 & 0 \end{bmatrix}$.

5. Solve the system $X' = AX$, where $A = \begin{bmatrix} -1 & -8 & 5 \\ 3 & 10 & -5 \\ 3 & 8 & -3 \end{bmatrix}$, given that the eigenvalues of A are $2, 2, 2$.

6. (a) Find all values of k for which the matrix $\begin{bmatrix} k & -1 & 0 \\ -1 & 1 & 1 \\ -2 & 1 & -k \end{bmatrix}$ is not invertible.

(b) Let $A = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 1 \\ -2 & 1 & 0 \end{bmatrix}$. Use Cayley-Hamilton theorem to find the inverse of A and the inverse of A^2, if they exist.

7. Let v_1, v_2, v_3 be linearly independent vectors in $\mathbb{R}^n$. Are the vectors:

$$w_1 = 4v_1 + 2v_3,\ w_2 = v_1 - 3v_2 + 2v_3,\ w_3 = 4v_2 - 2v_3$$

linearly dependent or independent? Justify.

8. An object is placed in a 350 °F oven at 1:00 pm. At 1:01 pm the temperature of the object is 230 °F and at 1:02 pm its temperature is 260 °F . The initial temperature of the object is:

(a) 170 °F

(b) 180 °F

(c) 190 °F

(d) 200 °F

(e) 210 °F

9. If y is a solution of the IVP: $x^2y' - xy = -y^2$, $y(1) = 1$, then $y(e) =$

(a) $2e$

(b) e

(c) $\frac{2e}{3}$

(d) $\frac{e}{2}$

(e) $\frac{e}{3}$

10. If a 3×3 matrix A has eigenvalues $2, 3,$ and -6, then the sum of the eigenvalues of A^{-1} is:

(a) $\frac{3}{2}$

(b) $-\frac{3}{2}$

(c) $\frac{2}{3}$

(d) $-\frac{2}{3}$

(e) $\frac{1}{3}$

11. If $A = \begin{bmatrix} 1 & 0 \\ -2 & 2 \end{bmatrix}$, then the sum of all the entries of A^{100} is:

(a) $3 - 2^{100}$

(b) $2^{100} - 3$

(c) $2^{100} - 1$

(d) $1 - 2^{100}$

(e) $2 - 2^{100}$

12. Which one of the following statements is TRUE for the given vectors?

(a) $(1, -2, 0)$, $(3, -2, 4)$, $(5, -2, 8)$ form a basis of $\mathbb{R}^3$.

(b) $(1, 1, 1, 2)$, $(2, 1, 1, 1)$, $(1, 2, 1, 1)$ span $\mathbb{R}^4$.

(c) $(1, 1, 0, 1)$, $(1, 0, 0, 1)$, $(1, 1, 1, 1)$ are linearly dependent.

(d) $(1, 0, 1, 0)$, $(0, 1, 0, 1)$, $(1, 0, 0, 1)$ form a basis of $\mathbb{R}^4$.

(e) $(1, 0, 1)$, $(2, 1, 0)$, $(3, 1, 1)$, $(4, 1, 1)$ span $\mathbb{R}^3$.