King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math 202
Final Exam, Third Semester (153), 2016
Net Time Allowed: 180 minutes

Name:——————————————————

ID:——————————Section:——————Serial:——————

Q#	Marks	Maximum Marks
1		12
2		10
3		8
4		8
5		10
6		10
7		10
8		10
9		10
10		12
11		10
12		12
13		10
14		8
Total		140

1. Write clearly.
2. Show all your steps.
3. No credit will be given to wrong steps.
4. Do not do messy work.
5. Calculators and mobile phones are NOT allowed in this exam.
6. Turn off your mobile.

1. (12 points) Use a suitable substitution to change the following DE to linear DE or separable (Do not the solve the new equation.

a. $2y' + (\tan x)y - \frac{4x+5}{\cos x}y^3 = 0$.

b. $x\frac{dy}{dx} = y(\ln y - \ln x)$.

2. (10 points) Solve the IVP

$$(4x^3 e^{x+y} + x^4 e^{x+y} + 2x) + \left(x^4 e^{x+y} + 2y\right) \frac{dy}{dx} = 0, \quad y(0) = 1.$$

3. (8 points) Find a homogenous Cauchy-Euler differential equation whose general solution is $y = c_1x^2 + c_2x^4$.

4. (8 points) Classify the singular points of

$$(1 - x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + \alpha(\alpha + 1)y = 0,$$

where α is a constant.

5. (10) Use the method of undetermined coefficients (Annihilator approach) to find the form of a particular solution of the differential equation

$$y'' + y' - 6y = 2\sin x \cos x + x + e^{2x}.$$

(Do not evaluate the coefficients).

6. (10 points) Solve $X' = \begin{bmatrix} -7 & 0 & 6 \\ 0 & 5 & 0 \\ 6 & 0 & 2 \end{bmatrix} X.$

7. (10 points) Three solutions of the equation $X' = AX$ are

$$\begin{bmatrix} e^t + e^{2t} \\ e^{2t} \\ 0 \end{bmatrix}, \quad \begin{bmatrix} e^t + e^{3t} \\ e^{3t} \\ e^{3t} \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} e^t - e^{3t} \\ -e^{3t} \\ -e^{3t} \end{bmatrix},$$

find the eigenvalues and eigenvectors of A.

8. (10 points) Solve the initial-value problem

$$X' = \begin{bmatrix} 1 & -2 \\ 2 & -3 \end{bmatrix} X, \quad X(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

9. (10 points) Solve $X' = \begin{bmatrix} -3 & 0 & 2 \\ 1 & -1 & 0 \\ -2 & -1 & 0 \end{bmatrix} X$, given that $\lambda_{1,2} = -1 \pm i\sqrt{2}$ and $\lambda_3 = -2$.

10. (12 points) Find a recurance relation for the power series solutions of the differential equation $(1-x)y'' + y' + (1-x)y = 0$ about $x_0 = 0$

    (Do not solve the equation)

11. (10 points) Using the substitution $y = \sum_{n=0}^{\infty} c_n x^n$ in a differential equation about the ordinary point $x = 0$, we get the relation

$$-2c_2 + (c_0 + 3c_1 - 6c_3)x + \sum_{k=2}^{\infty}[-(k+2)(k+1)c_{k+2} + k(k+2)c_k + c_{k-1}]x^k = 0.$$

Find two power series solutions of the differential equation.

12. (12 points) Use the method of Frobenius to find a series solution of the differential equation $x^2y'' + (x^2 - 3x)y' + 3y = 0$ about the regular singular point $x = 0$ using the largest indicial root.

13. (10 points) Let $X = c_1 \begin{pmatrix} 4 \\ 1 \end{pmatrix} e^{3t} + c_2 \begin{pmatrix} 2 \\ -1 \end{pmatrix} e^{-3t}$ be the general solution of the homogenous system $X' = AX$. Find a particular solution of $X' = AX + \begin{pmatrix} 6e^{3t} \\ 0 \end{pmatrix}$.

14. (8 points) Use matrix exponential to solve the homogenous system $X' = \begin{pmatrix} -2 & 0 \\ 0 & 5 \end{pmatrix} X$ subject to $X(0) = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$.