King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math 202
Exam II, Third Semester (153), 2016
Net Time Allowed: 120 minutes

Name:————————————————————————————————————

ID:————————————————Section:——————————Serial:————————————

Q#	Marks	Maximum Marks
1		7
2		5
3		5
4		8
5		11
6		10
7		10
8		14
9		10
10		10
11		10
Total		100

1. Write clearly.

2. Show all your steps.

3. No credit will be given to wrong steps.

4. Do not do messy work.

5. Calculators and mobile phones are NOT allowed in this exam.

6. Turn off your mobile.

1. (7 points) Show that $\{\cos(\ln x), \sin(\ln x)\}$ form a fundamental set for the differential equation $x^2 y'' + xy' + y = 0$. Find the general solution.

2. (5 points) Without the use of Wronskian determine whether the set $\{1, \cos^2 x, \cos 2x\}$ is linearly dependent or linearly idenpendent over $(-\infty, \infty)$

3. (5 points) Assume $y'' - 6y' + 5y = -9e^{2x}$ has a particular solution $y_{p_1} = 3e^x$ and $y'' - 6y' + 5y = 5x^2 + 3x - 16$ has a particular solution $y_{p_2} = x^2 + 3x$. Find the solution of $y'' - 6y' + 5y = 27e^{2x} + 10x^2 + 6x - 32$

4. (10 points) Find a linear differential operator with the least order which annihilates

$$xe^{-2x}\sin 3x + x^2 - 3 + 2\cos x \cos 3x.$$

5. (11 points) Given that $y_1 = e^x \sin 2x$ is a solution of

$$\frac{d^4y}{dx^4} + 3\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} + 13\frac{dy}{dx} + 30y = 0.$$

Find the general solution of the given DE.

6. (10 points) Find the general solution of

$$(\sin^2 x)\frac{d^2y}{dx^2} - (2\sin x \cos x)\frac{dy}{dx} + (\cos^2 x + 1)y = \sin^3 x$$

given that $y_1 = \sin x$ and $y_2 = x\sin x$ are linearly independent solutions of the corresponding homogeneous DE.

7. (10 points) Solve the boundary value problem

$$9\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 5y = 0, \quad y(0) = 6, \ y(\frac{3\pi}{4}) = 1.$$

8. (14 points) Solve the initial value problem

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 4y = 16x + 20e^x, \quad y(0) = 0, \ y'(0) = 3.$$

9. (10 points) Given that the equation

$$t\frac{d^2y}{dt^2} - (1+3t)\frac{dy}{dt} + 3y = 0, \quad t > 0,$$

has a solution of the form e^{ct}, for some constant c, find the general solution.

10. (10 points) Use a suitable substitution to transform the DE:

$$x^2 \frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + 6y = \ln x^2, \quad x > 0,$$

to a DE with constant coefficients (Do not solve the new equation).

11. (10 points) Solve

$$(2x - 3)^2 \frac{d^2y}{dx^2} - 6(2x - 3)\frac{dy}{dx} + 12y = 0, \quad x \in (3/2, \infty).$$