King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math 202
Exam I, Third Semester (153), 2016
Net Time Allowed: 120 minutes

Name:______________________________

ID:______________Section:__________Serial:__________

Q#	Marks	Maximum Marks
1		10
2		12
3		6
4		8
5		10
6		10
7		12
8		6
9		6
10		10
11		10
Total		100

1. Write clearly.
2. Show all your steps.
3. No credit will be given to wrong steps.
4. Do not do messy work.
5. Calculators and mobile phones are NOT allowed in this exam.
6. Turn off your mobile.

1. (10 points) Show that $\ln\left(\frac{2x-1}{x-1}\right) = t$ is an implicit solution of

$$\frac{dx}{dt} = (x-1)(1-2x)$$

write the explicit solution and give the interval I of definition of the solution.

2. (12 points) Consider the following differential equation

$$\frac{dy}{dx} = (2 - y)(y - 4)$$

(a) Solve the differential equation

(b) Find a singular solution if it exists for the differential equation

3. (6 points) Find and sketch the region where the initial-value problem $y' = x\sqrt{y}, \quad y(x_0) = y_0$ has a unique solution at every (x_0, y_0).

4. (8 points) Use a suitable substitution to change the following differential equation into linear first order differential equation (DO NOT SOLVE THE NEW EQUATION)

$$\frac{dy}{dx} = y\cot x + y^3 \csc x$$

5. (10 points) Solve the initial value problem

$$(x+3)\frac{dy}{dx} + (2x+1)y = e^{-2x}(x+3)^7 \sin x, \quad y\left(\frac{\pi}{2}\right) = 0$$

6. (10 points) Solve: $\dfrac{dy}{dx} = \sin(x + y)$

7. (12 points) Show that the following differential equation is exact and solve it

$$(y^2 \cos x - 3x^2 y - 2x)dx = (x^3 - 2y \sin x - \ln y)dy$$

8. (6 points) Find an appropriate integrating factor to change the following differential equation into EXACT differential equation (Do not solve the differential equation)

$$(y^2 + xy^3)dx + (5y^2 - xy + y^3 \sin y)dy = 0$$

9. (6 points) Write the homogeneous differential equation

$$(\sqrt{x+y} + \sqrt{x-y})dx + (\sqrt{x-y} - \sqrt{x+y})dy = 0$$

into a separable differential equation using a suitable substitution. Find the new equation (Do not solve it).

10. (10 points) Radium decomposes at a rate proportional to the amount present at a time t. If 20% of the original amount P_0 disappears in 700 years, find the amount present after 1400 years.

11. (10 points) Show that $y = \frac{-\sin 2x}{3} + c_1 \sin x + c_2 \cos x$ is a two-parameter family of solutions to the differential equation $y'' + y = \sin 2x$. Determine whether a member of the family can be found that satisfies the boundary conditions $y(0) = 0$ and $y(\pi) = 0$.