King Fahd University of Petroleum & Minerals

Department of Mathematics and Statistics

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Math 201: Final Exam, Summer 153 (180 minutes) — MASTER

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Name: **Student ID:** **Section:** **Serial:**

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Part II		Grade	
Q1			/14
Q2			/14
Q3			/10
Q4			/10
Q5			/12
Q6			/10
TOTAL			/70

Part I. Choose the correct answer and shade the corresponding bubble in the OMR Sheet (no partial credit):

1. The length of the parametric curve

$$x = \sin^2(t),\ y = \cos^2(t),\ 0 \leq t \leq \pi$$

is equal to

(a) $2\sqrt{2}$

(b) 4

(c) $2 - 2\sqrt{2}$

(d) $3 + 2\sqrt{2}$

(e) 1

2. The slope of the tangent line to the polar curve $r = e^{-\theta}$ at the point corresponding to $\theta = \pi$ is

(a) -1

(b) 0

(c) 2

(d) -2

(e) $\frac{1}{2}$

3. The function $f(x,y) = xy + \frac{1}{x} + \frac{1}{y}$ has one critical point (a,b) and

(a) f has a local minimum at (a,b)

(b) f has a local maximum at (a,b)

(c) f has a saddle point at (a,b)

(d) $f(a,b) = 5$

(e) $f_{xy}(a,b) = 0$

4. $\int_0^1 \int_0^1 \frac{y}{1+xy}\, dy\, dx =$

(a) $2\ln 2 - 1$

(b) $2\ln 2 - 3$

(c) $\ln 2$

(d) $1+\sqrt{2}$

(e) $\ln 2 + 1$

5. The surface consisting of all points $P(x, y, z)$ such that the distance from P to the plane $y = 1$ is equal to the distance from P to the point $(0, -1, 0)$ is a

(a) paraboloid
(b) cone
(c) hyperboloid of one sheet
(d) hyperboloid of two sheets
(e) sphere

6. $\lim_{(x,y)\to(0,0)} \tan(x)\sin(\frac{|x|}{|x|+|y|})$

(a) equals 0
(b) equals $\sin\frac{1}{2}$
(c) equals 1
(d) equals 2
(e) does not exist

7. The point on the graph of $z = 3x^2 - 4y^2$ at which the vector $\vec{n} =< 3, 2, 2, >$ is normal to the tangent plane is

(a) $(-\frac{1}{4}, \frac{1}{8}, \frac{1}{8})$

(b) $(\frac{1}{2}, -\frac{1}{2}, -\frac{1}{4})$

(c) $(0, \frac{1}{2}, -1)$

(d) $(\frac{1}{\sqrt{3}}, \frac{1}{2}, 0)$

(e) $(\frac{1}{4}, -\frac{1}{8}, \frac{1}{8})$

8. If $P = \sqrt{u^2 + v^2 + w^2}$, where

$$u = xe^y,\ v = ye^x \text{ and } w = e^{xy},$$

then the value of $\frac{\partial P}{\partial x}$ at $x = 0,\ y = 2$ is

(a) $\frac{6}{\sqrt{5}}$

(b) $\frac{1}{\sqrt{5}}$

(c) $\sqrt{5}$

(d) $3\sqrt{5}$

(e) $\frac{-2}{\sqrt{5}}$

9. Let $\overrightarrow{u}$ be the unit vector in which the directional derivative of $f(x, y, z) = \frac{x+y}{z}$ at $P(1, -1, \frac{1}{2})$ attains its minimum value. If $\overrightarrow{v} =< 3\sqrt{2}, -2\sqrt{2}, -\sqrt{2} >$, then $\overrightarrow{u} \cdot \overrightarrow{v} =$

(a) -1

(b) 0

(c) 1

(d) 5

(e) -5

10. The length of the line segment of the line

$$L : x = 1 - t,\ y = t,\ z = 2 + t,\ t \in \mathbb{R}$$

that is contained inside the sphere$(x-1)^2+(y+2)^2+(z-1)^2 = 5$ is

(a) $2\sqrt{3}$

(b) 2

(c) $\sqrt{3}$

(d) 3

(e) $\sqrt{6}$

11. Let $\overrightarrow{u}$ and $\overrightarrow{v}$ be two unit vectors with an angle of $\frac{\pi}{3}$ between them. Then $(\overrightarrow{u} + \text{proj}_{\overrightarrow{v}} \overrightarrow{u}) \cdot \text{proj}_{\overrightarrow{u}} \overrightarrow{v}$ equals

(a) $\frac{5}{8}$
(b) 1
(c) 0
(d) $\frac{3}{8}$
(e) 2

12. The following equations (in the spherical coordinates)

$$\rho = 4, \ \phi = \frac{\pi}{4}, \ \theta = \frac{\pi}{3}$$

represent respectively

(a) sphere, half cone, half plane
(b) sphere, plane, cone
(c) sphere, cone, half cone
(d) sphere, half plane, plane
(e) sphere, line, line

13. $$\int_0^2 \int_{-\sqrt{2y-y^2}}^{\sqrt{2y-y^2}} \int_{x^2+y^2}^{2y} dz\ dx\ dy =$$

(a) $$\int_{-1}^{1} \int_{1-\sqrt{1-x^2}}^{1+\sqrt{1-x^2}} \int_{x^2+y^2}^{2y} dz\ dy\ dx$$

(b) $$\int_0^2 \int_{x^2+y^2}^{2y} \int_{-\sqrt{2y-y^2}}^{\sqrt{2y-y^2}} dx\ dz\ dy$$

(c) $$\int_{-1}^{1} \int_0^1 \int_{x^2+y^2}^{2y} dz\ dy\ dx$$

(d) $$\int_0^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{x^2+y^2}^{2y} dz\ dy\ dx$$

(e) $$\int_0^2 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \int_{x^2+y^2}^{2y} dz\ dx\ dy$$

14. $$\int_{-1}^{1}\int_{0}^{\sqrt{1-y^2}}\int_{0}^{x}(x^2+y^2)\ dz\ dx\ dy =$$

(a) $$\int_{-\pi/2}^{\pi/2}\int_{0}^{1}\int_{0}^{r\cos\theta} r^3\ dz\ dr\ d\theta$$

(b) $$\int_{0}^{\pi}\int_{0}^{1}\int_{0}^{r\cos\theta} r^2\ dz\ dr\ d\theta$$

(c) $$\int_{-\pi/2}^{\pi/2}\int_{0}^{1}\int_{-1}^{r\cos\theta} r^2\ dz\ dr\ d\theta$$

(d) $$\int_{-\pi}^{\pi}\int_{0}^{1}\int_{-1}^{1} r^3\ dz\ dr\ d\theta$$

(e) $$\int_{0}^{r\cos\theta}\int_{0}^{\pi}\int_{0}^{1} r^3\ dr\ d\theta\ dz$$

Part II. Solve each of the following 6 questions showing full details.

Q1. (14 Points) Find the absolute maximum and the absolute minimum of

$$f(x, y) = 2x + 4y - x^2 - 2y^2$$

on the rectangular region $R = \{(x, y) : 0 \leq x \leq 4,\ 0 \leq y \leq 3\}$.

Q2. (14 Points) Find the point(s) closest to the origin on the line of intersection of the planes

$$3x - 2y + z = 5 \text{ and } 2x + y - 2z = -8.$$

Q3. (10 Points) Find the volume of the solid between the two planes

$$x + y + z = 2,\ z - 2x - 2y = 10$$

that lies above region in the xy-plane bounded by

$$y = 1 - x^2,\ y = x^2 - 1.$$

Q4. (10 Points) Convert the integral

$$I = \int_0^2 \int_0^{\sqrt{2x-x^2}} \sqrt{x^2+y^2}\,dy\,dx$$

to polar coordinates, then evaluate it.

Q5. (12 Points) Find the volume of the solid bounded by the cylinder $x^2 + y^2 = 9$ and the planes $z = 0$ and $x + z = 3$.

Q6. (10 Points) Find the volume of the solid below the sphere $x^2 + y^2 + z^2 = 2z$ and above the cone $z = \sqrt{x^2 + y^2}$.