1. Using three rectangles and midpoints, the area under the graph of $f(x) = x + \cos x$ from $x = 0$ to $x = \pi$ is estimated as

$\pi/6 \quad \pi/2 \quad \frac{5\pi}{6}$

$0 \quad \pi/3 \quad \frac{2\pi}{3} \quad \pi$

(a) $\dfrac{\pi^2}{2}$

(b) $\dfrac{\pi^2}{3}$

(c) $\dfrac{\pi^3}{3}$

(d) $\dfrac{\pi^3}{6}$

(e) π^2

$$\Delta x = \frac{\pi - 0}{3} = \pi/3$$

$$A = \frac{\pi}{3}\left[\frac{\pi}{6} + \cos\frac{\pi}{6} + \frac{\pi}{2} + \cos\frac{\pi}{2} + \frac{5\pi}{6} + \cos\frac{5\pi}{6}\right]$$

$$= \frac{\pi}{3}\left[\frac{3\pi}{2} + \frac{\sqrt{3}}{2} + 0 - \frac{\sqrt{3}}{2}\right]$$

$$= \frac{\pi}{3} \cdot \frac{3\pi}{2} = \frac{\pi^2}{2}$$

2. If R_n is the Riemann sum for $f(x) = 4 + \dfrac{x^2}{8}$; $0 \le x \le 4$ with n subintervals and taking sample points to be the right end points, then $R_n =$

(a) $16 + \dfrac{4(n+1)(2n+1)}{3n^2}$

(b) $4 + \dfrac{2(n+1)(2n+1)}{3n^2}$

(c) $16 + \dfrac{16(n+1)(2n+1)}{n^2}$

(d) $16 + (n+1)(2n+1)$

(e) $16 + \dfrac{4}{3}n(n+1)(2n+1)$

$$\Delta x = \frac{4-0}{n} = \frac{4}{n}$$

$$x_i = 0 + \frac{4i}{n}$$

$$R_n = \sum_{i=1}^{n} f(x_i)\,\Delta x = \sum_{i=1}^{n}\left[4 + \frac{16 i^2}{8n^2}\right]\frac{4}{n}$$

$$= \sum_{i=1}^{n}\left[\frac{16}{n} + \frac{8i^2}{n^3}\right] = \frac{16}{n}\cdot n + \frac{8}{n^3}\,\frac{n(n+1)(2n+1)}{6}$$

$$= 16 + \frac{4(n+1)(2n+1)}{3n^2}$$

3. $\int_0^1 (x^3 + 3^x)\,dx =$

(a) $\frac{1}{4} + \frac{2}{\ln 3}$

(b) $\frac{\ln 3}{4}$

(c) $\frac{1}{2} + \frac{2}{\ln 3}$

(d) $\frac{2}{\ln 3}$

(e) $\frac{\ln 4}{3}$

$$\left[\frac{x^4}{4} + \frac{3^x}{\ln 3} \right]_0^1$$

$$= \frac{1}{4} + \frac{3}{\ln 3} - \frac{1}{\ln 3}$$

$$= \frac{1}{4} + \frac{2}{\ln 3}$$

4. $\int_0^{\pi/3} (\sin x \ \sec^2 x)\,dx =$

(a) 1

(b) $\sqrt{3}$

(c) $\sqrt{3}/2 - 1$

(d) -1

(e) $2/\sqrt{3} - 1$

$$\int_0^{\pi/3} \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x}\,dx$$

$$= \int_0^{\pi/3} \tan x \sec x \,dx$$

$$= \Big[\sec x\Big]_0^{\pi/3}$$

$$= \sec \frac{\pi}{3} - \sec 0$$

$$= 2 - 1$$

$$= 1$$

5. The value of the integral $\int_{-\pi/3}^{\pi/3} (x \tan x^4 + x^2)\, dx$ is

(a) $\frac{2\pi^3}{81}$

(b) $\frac{2\pi^2}{81}$

(c) $\frac{\pi^3}{27}$

(d) $\frac{\pi^2}{27}$

(e) $\frac{3\pi^2}{16}$

$= \int_{-\pi/3}^{\pi/3} x \tan x^4 \, dx + \int_{-\pi/3}^{\pi/3} x^2 \, dx$

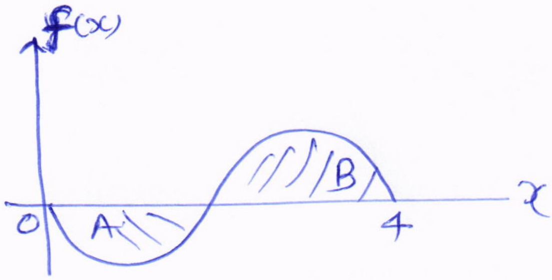
$= 0 + 2\int_0^{\pi/3} x^2 \, dx$

(odd function)

$= 2\left[\frac{x^3}{3}\right]_0^{\pi/3} = \frac{2}{3} \cdot \frac{\pi^3}{27}$

$= \frac{2\pi^3}{81}$

6. In the figure shown, regions A and B are bounded by the graph of a function f and the x-axis. If the area of region A is $\frac{1}{6}$ and the area of the region B is $\frac{3}{2}$, then $\int_0^4 f(x)\, dx + \int_0^4 |f(x)|\, dx =$



(a) 3

(b) 1

(c) 2

(d) $\frac{5}{3}$

(e) $\frac{-4}{3}$

$= \left(-\frac{1}{6} + \frac{3}{2}\right) + \left(\frac{1}{6} + \frac{3}{2}\right)$

$= \frac{3}{2} + \frac{3}{2} = 3$

7. If $\int_{-1}^{1} f(x)\,dx = 3$, $\int_{1}^{3} f(x)\,dx = 4$ and $\int_{-1}^{0} f(x)\,dx = 3$, then $\int_{0}^{3} f(x)\,dx =$

$$\int_0^{-1} f(x)\,dx + \int_{-1}^{3} f(x)\,dx$$

$$= -3 + \int_{-1}^{1} f(x)\,dx + \int_{1}^{3} f(x)\,dx$$

$$= -3 + 3 + 4$$

$$= 4$$

(a) 4

(b) 5

(c) 3

(d) 2

(e) 1

8. If $f(x) = \begin{cases} -x; & -4 \le x < 0 \\ \sqrt{4 - x^2}; & 0 \le x \le 2 \end{cases}$, then evaluate the integral $\int_{-4}^{2} f(x)\,dx$ by interpreting in terms of area(s).

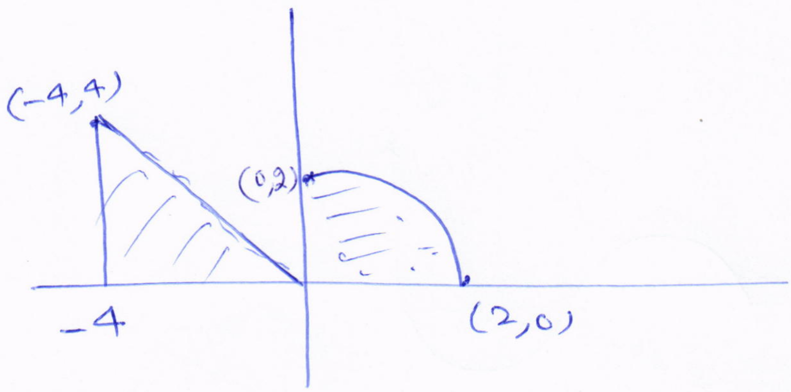
(a) $\pi + 8$

(b) $\pi + 5$

(c) π

(d) $\frac{\pi}{2} + 8$

(e) $\frac{\pi}{2} + 3$



$$A = \frac{1}{2}(+4)(4) + \frac{1}{4} \cdot \pi \cdot 4$$

$$= \pi + 8$$

9. $\displaystyle\int_0^{1/2} \frac{\cos(\sin^{-1} x)}{\sqrt{1-x^2}}\, dx =$

(a) $\frac{1}{2}$

(b) 1

(c) $\frac{3}{2}$

(d) 0

(e) $-\frac{3}{2}$

Let $\sin^{-1} x = u$. Then $\frac{1}{\sqrt{1-x^2}}\,dx = du$

$x=0, u=0$
$x=\frac{1}{2}, u=\frac{\pi}{6}$

$\int \cos u \, du = \left[\sin u\right]_0^{\pi/6}$

$= \sin\frac{\pi}{6} - \sin 0$

$= \frac{1}{2}$

10. $\displaystyle\int \sqrt[3]{x^3+1} \cdot x^5 \, dx =$

(a) $\frac{1}{7}(x^3+1)^{7/3} - \frac{1}{4}(x^3+1)^{4/3} + C$

(b) $\frac{1}{7}(x^3+1)^{1/3} + \frac{1}{4}(x^3+1)^{1/3} + C$

(c) $(x^3+1)^{7/3} - (x^3+1)^{4/3} + C$

(d) $\frac{1}{4}(x^3+1)^{7/3} - \frac{1}{7}(x^3+1)^{4/3} + C$

(e) $(x^3+1)^{1/3} + (x^3+1)^{2/3} + C$

Let $u = x^3+1$. Then
$du = 3x^2 dx$
and $x^3 = u-1$

$\int (x^3+1)^{1/3} \cdot x^3 \cdot x^2 dx$

$= \int u^{1/3} (u-1) \frac{1}{3} du = \frac{1}{3}\int (u^{\frac{4}{3}} - u^{1/3}) du$

$= \frac{1}{3}\left[\frac{3}{7}u^{7/3} - \frac{3}{4}u^{4/3}\right] + C$

$= \frac{1}{7}(x^3+1)^{7/3} - \frac{1}{4}(x^3+1)^{4/3} + C$

11. $\int_0^{\pi/4} (1+\tan\theta)^3 \sec^2\theta \, d\theta =$

(a) $\frac{15}{4}$

(b) $\frac{1}{4}$

(c) $\frac{15}{2}$

(d) $\frac{11}{4}$

(e) $\frac{13}{4}$

Let $1+\tan\theta = u$. Then $\sec^2\theta \, d\theta = du$.

and $\theta = 0 \Rightarrow u = 1$

$\theta = \pi/4 \Rightarrow u = 2$

$$I = \int_1^2 u^3 \, du = \left[\frac{u^4}{4}\right]_1^2 = 4 - \frac{1}{4}$$

$$= \frac{15}{4}$$

12. The area of the region enclosed by the curves $y = x^2 - 2x$ and $y = 2 - x$ is

(a) $\frac{9}{2}$

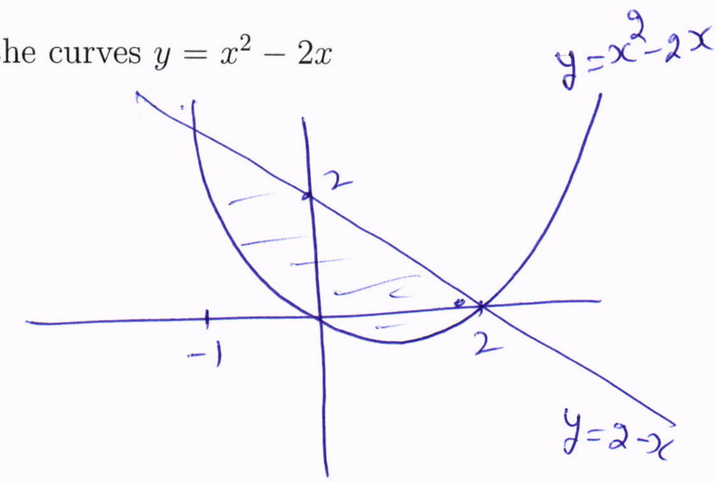
(b) $\frac{5}{2}$

(c) 0

(d) 2

(e) 1

Points of intersections are -1 and 2



$$A = \int_{-1}^{2} (2 - x - x^2 + 2x) \, dx$$

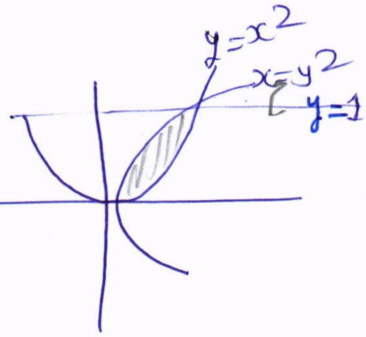
$$= \int_{-1}^{2} (2 + x - x^2) \, dx = \left[2x + \frac{x^2}{2} - \frac{x^3}{3}\right]_{-1}^{2}$$

$$= \left(4 + \frac{4}{2} - \frac{8}{3}\right) - \left(-2 + \frac{1}{2} + \frac{1}{3}\right)$$

$$= \left(6 - \frac{8}{3}\right) - \left(-\frac{3}{2} + \frac{1}{3}\right)$$

$$= \frac{10}{3} + \frac{7}{6} = \frac{20+7}{6} = \frac{27}{6} = \frac{9}{2}$$

13. The volume of the solid obtained by rotating the region bounded by the curves $y = x^2$ and $x = y^2$ about $y = 1$ is



(a) $\frac{11\pi}{30}$

(b) $\frac{3\pi}{4}$

(c) $\frac{15\pi}{7}$

(d) $\frac{19\pi}{35}$

(e) $\frac{5\pi}{7}$

$$V = \pi \int_0^1 \left[(1-x^2)^2 - (1-\sqrt{x})^2 \right] dx$$

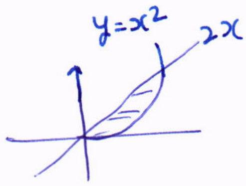
$$= \pi \int_0^1 \left[(1 + x^4 - 2x^2) - (1 + x - 2\sqrt{x}) \right] dx$$

$$= \pi \left[\frac{x^5}{5} - \frac{2}{3}x^3 - \frac{x^2}{2} + \frac{2}{3}\cdot 2x^{3/2} \right]_0^1$$

$$= \pi \left(\frac{1}{5} - \frac{2}{3} - \frac{1}{2} + \frac{4}{3} \right)$$

$$= \pi \, \frac{6 - 20 - 15 + 40}{30} = \frac{11\pi}{30}$$

14. The volume of the solid obtained by rotating the region enclosed by the curves $y = 2x$, $y = x^2$ about x-axis is



(a) $\frac{64\pi}{15}$

(b) $\frac{2\pi}{15}$

(c) 2π

(d) $\frac{\pi}{3}$

(e) $\frac{2\pi}{27}$

$$V = \int_0^2 \pi (4x^2 - x^4) dx$$

$$= \pi \left[\frac{4}{3}x^3 - \frac{x^5}{5} \right]_0^2$$

$$= \pi \left[\frac{32}{3} - \frac{32}{5} \right]$$

$$= 32\pi \left[\frac{1}{3} - \frac{1}{5} \right]$$

$$= \frac{64\pi}{15}$$

15. Consider a solid with a circular base of radius 1. Parallel cross-sections perpendicular to the base are squares. The volume of the solid is

(a) $\frac{16}{3}$

(b) $\frac{8}{9}$

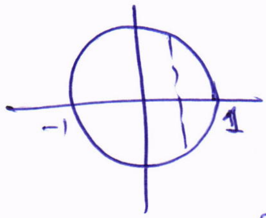
(c) $\frac{1}{9}$

(d) $\frac{5}{3}$

(e) $\frac{2}{3}$

one side of the square $= 2\sqrt{1-x^2}$

$A = 4(1-x^2)$



$x^2+y^2=1$

$$V = \int_{-1}^{1} 4(1-x^2)\,dx$$

$$= 8\int_0^1 (1-x^2)\,dx = 8\left[x-\frac{x^3}{3}\right]_0^1$$

$$= 8\left[1-\frac{1}{3}\right]$$

$$= 8\cdot\frac{2}{3} = \frac{16}{3}$$

16. $\lim_{n\to\infty}\sum_{i=1}^{n}\left[1+\sin\left(1+\frac{i}{n}\right)\right]\frac{2}{n} =$

(a) $\int_2^4 \left[1+\sin\left(\frac{x}{2}\right)\right] dx$

(b) $\int_2^4 \left[1+\sin\left(\frac{x+1}{2}\right)\right] dx$

(c) $\int_1^3 \left[1+\sin\left(\frac{x}{2}\right)\right] dx$

(d) $\int_0^2 \left[2+\sin\left(\frac{x+2}{2}\right)\right] dx$

(e) $\int_0^2 \left[1+\sin\left(\frac{x}{2}\right)\right] dx$

$\Delta x = \frac{2}{n}$

$x_i = a + \frac{2i}{n}$

$\Rightarrow \frac{i}{n} = \frac{x_i - a}{2}$

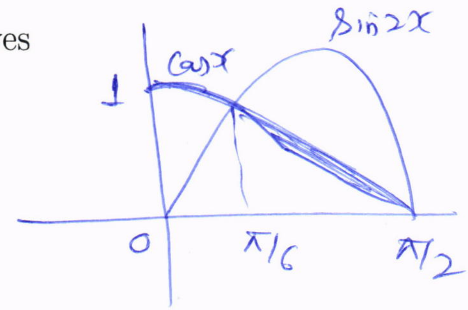
and

$1+\frac{i}{n} = \frac{x_i - a + 2}{2}$

For $a = 2$, $1+\frac{i}{n} = \frac{x_i}{2}$

Ans (a)

17. The area of the region bounded by the curves $y = \cos x$, $y = \sin 2x$, $x = 0$ and $x = \dfrac{\pi}{2}$ is



(a) $\dfrac{1}{2}$

(b) $\dfrac{1}{6}$

(c) 1

(d) 2

(e) $\dfrac{2}{3}$

$$A = \int_0^{\pi/6} (\cos x - \sin 2x)\,dx + \int_{\pi/6}^{\pi/2} (\sin 2x - \cos x)\,dx$$

$$= \left[\sin x + \tfrac{1}{2}\cos 2x\right]_0^{\pi/6} + \left[-\tfrac{1}{2}\cos 2x - \sin x\right]_{\pi/6}^{\pi/2}$$

$$= \cancel{\tfrac{1}{2}} + \tfrac{1}{4} - (0 + \cancel{\tfrac{1}{2}}) + (\tfrac{1}{2} - 1) - (-\tfrac{1}{4} - \tfrac{1}{2})$$

$$= \tfrac{1}{4} - \cancel{\tfrac{1}{2}} + \tfrac{1}{4} + \cancel{\tfrac{1}{2}}$$

$$= \tfrac{1}{2}$$

18. The volume of the solid obtained by rotating the region bounded by the curves $y = \sqrt{x}$, $y = 2x - 1$ and $y = 0$ about $y-$axis is

(a) $\dfrac{23}{60}\pi$

(b) $\pi/2$

(c) 2π

(d) π

(e) $\pi/6$

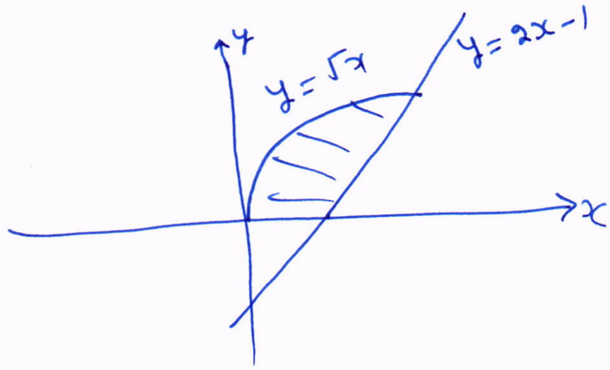
$$V = \pi \int_0^1 \left[\left[\frac{y+1}{2} - 0\right]^2 - (y^2 - 0)^2\right] dy$$

$$= \pi \int_0^1 \left(\frac{y^2 + 2y + 1}{4} - y^4\right) dy$$

$$= \pi \left[\frac{y^3}{12} + \frac{y^2}{4} + \frac{y}{4} - \frac{y^5}{5}\right]_0^1$$

$$= \pi \left(\frac{1}{12} + \frac{1}{4} + \frac{1}{4} - \frac{1}{5}\right)$$

$$= \pi \quad \frac{5 + 15 + 15 - 12}{60} = \frac{23\pi}{60}$$



19. A particle moves along a line such that the velocity at time t (seconds) is $v(t) = \cos t$ (meters per seconds). Find the total distance travelled during $0 \le t \le 2\pi$.

(a) $4\,meters$

(b) $3\,meters$

(c) $2\,meters$

(d) $1\,meter$

(e) $0\,meters$

$$\text{Total distance} = \int_0^{2\pi} |\cos t|\,dt$$

$$= \int_0^{\pi/2} \cos t\,dt - \int_{\pi/2}^{3\pi/2} \cos t\,dt + \int_{3\pi/2}^{2\pi} \cos t\,dt$$

$$= \sin t\Big|_0^{\pi/2} - \sin t\Big|_{\pi/2}^{3\pi/2} + \sin t\Big|_{3\pi/2}^{2\pi}$$

$$= (1-0) - (-1-1) + (0-(-1))$$

$$= 1 + 2 + 1 = 4 \text{ meters}$$

20. If f is a continuous function and $\int_0^x f(t)\,dt = x\cos(\pi x)$, then $f(4) =$

(a) 1

(b) 2

(c) $\frac{1}{2}$

(d) 5

(e) $-\frac{1}{2}$

By FTC 1:

$$f(x) = \frac{d}{dx}\left[x\cos \pi x\right]$$

$$= \cos \pi x + x(-\sin \pi x)\cdot \pi$$

$$f(4) = \cos 4\pi - 4\pi \sin(4\pi)$$

$$= 1 - 0$$

$$= 1$$