Serial No.:______ Student Name:____________________________ Student Number: ______

Instructor: M. Z. Abu-Sbeih Math 101- Q2 Date: 8-8-2016

SHOW ALL YOUR WORK. NO CREDITS FOR ANSWERES WITHOUT JUSTIFICATIONS

Problem 1: (12 points) Evaluate the limit if it exists.

a) $\lim_{x\to 2} \frac{\sqrt[3]{4x}-2}{x-2}$ $= \lim_{x\to 2} \frac{f(x)-f(2)}{x-2}$

$f(x) = \sqrt[3]{4x}$

$f'(x) = \frac{4}{3}(4x)^{-\frac{2}{3}}$

$f(2) = \sqrt[3]{8} = 2.$

$= f'(2)$

$= \frac{4}{3}(8)^{-\frac{2}{3}}$

$= \frac{4}{3}\,\frac{1}{4} = \frac{1}{3}$

a) $\lim_{x\to 0} \frac{(3+x)^2 \tan 2x}{x}$ $= \lim_{x\to 0} (3+x)^2 \frac{\sin 2x}{x} \cdot \frac{1}{\cos 2x}$

$= \lim_{x\to 0} \frac{(3+x)^2}{\cos 2x} \frac{2\sin 2x}{2x}$

$= \frac{9}{1}(2) = 18$

Problem 2: (7 points) If $f(2)=-3,\ g(2)=4,\ f'(2)=-2,\ g'(2)=7$, find $h'(2)$ where

$h(x) = \frac{xg(x)}{1+f(x)}$

$h'(x) = \frac{[1+f(x)][g(x)+xg'(x)] - xg(x)f'(x)}{[1+f(x)]^2}$

$h'(2) = \frac{[1+f(2)][g(2)+2g'(2)] - 2g(2)f'(2)}{[1+f(2)]^2}$

$= \frac{-2[4+14] - 2(4)(-2)}{4}$

$= \frac{-36+16}{4} = -5$

Problem 3: **(7 points)** Find $f'(x)$ for the function and simplify your answer completely.

$$f(x)=\frac{\tan x\ -1}{\sec x}$$

$$f'(x)=\frac{\sec x\sec^2 x-(\tan x-1)(\sec x\tan x)}{\sec^2 x}$$

$$=\frac{\sec x\left[\sec^2 x-\tan^2 x+\sec x\tan x\right]}{\sec^2 x}$$

$$=\frac{1+\sec x\tan x}{\sec x}$$

Problem 4: **(7 points)** Find the slope of the line tangent to the curve $y=(\sin x)^{\ln x}$ at the point (1,1).

$$\ln y=\ln(\sin x)^{\ln x}=(\ln x)\ln(\sin x)$$

$$\frac{y'}{y}=\frac{\ln(\sin x)}{x}+\frac{\cos x}{\sin x}\ln x$$

$$y'\Big|_{(1,1)}=\left[\ln(\sin 1)+\left(\frac{\cos 1}{\sin 1}\right)(\ln 1)\right](\sin 1)^{(\ln 1)}$$

$$=\left[\ln(\sin 1)+(\tan 1)(0)\right](\sin 1)^0$$

$$=\ln(\sin 1)$$

Problem 5: **(7 points)** Find y' at the point (2,0) where

$$\tan^{-1}(xy)=xy^3$$

$$\frac{xy'+y}{1+x^2y^2}=3xy^2y'+y^3$$

at (2,0):

$$\frac{2y'+0}{1+0}=3(0)+0.$$

$$2y'=0$$

$$y'=0.$$