Serial No.:______ Student Name: Key Student Number: ______
Instructor: M. Z. Abu-Sbeih Math 101- Q1 Date: 21-7-2016

SHOW ALL YOUR WORK. NO CREDITS FOR ANSWERES WITHOUT JUSTIFICATIONS

Problem 3: (16 points) Find the limit if it exists

a) $\lim\limits_{x\to 2^-} \dfrac{x^2-2x}{x^2-4x+4} = \lim\limits_{x\to 2^-} \dfrac{x(x-2)}{(x-2)(x-2)} = \lim\limits_{x\to 2^-} \dfrac{x}{x-2} = -\infty$

b) $\lim\limits_{x\to -3} \dfrac{\sqrt{x^2+16}-5}{x+3} = \lim\limits_{x\to -3} \dfrac{\sqrt{x^2+16}-5}{x+3} \cdot \dfrac{\sqrt{x^2+16}+5}{\sqrt{x^2+16}+5}$

$= \lim\limits_{x\to -3} \dfrac{x^2+16-25}{(x+3)(\sqrt{x^2+16}+5)}$

$= \lim\limits_{x\to -3} \dfrac{(x-3)\cancel{(x+3)}}{\cancel{(x+3)}(\sqrt{x^2+16}+5)} = \dfrac{-6}{\sqrt{9+16}+5} = \dfrac{-3}{5}$

c) $\lim\limits_{x\to 0} (1-\cos x)\cos\dfrac{1}{x}$ By the Squeeze Theorem:

We know that $-1 \le \cos\dfrac{1}{x} \le 1$

But $1-\cos x \ge 0$, then $-(1-\cos x) \le (1-\cos x)\cos\dfrac{1}{x} \le (1-\cos x)$

Now: $\lim\limits_{x\to 0} -(1-\cos x) = \lim\limits_{x\to 0} (1-\cos x) = 0.$

$\therefore$ By Squeeze Theorem $\lim\limits_{x\to 0} (1-\cos x)\cos\dfrac{1}{x} = 0.$

d) $\lim\limits_{x\to 0^+} \tan^{-1}(x+\ln x) =$

Since $\tan^{-1}$ is continuous,

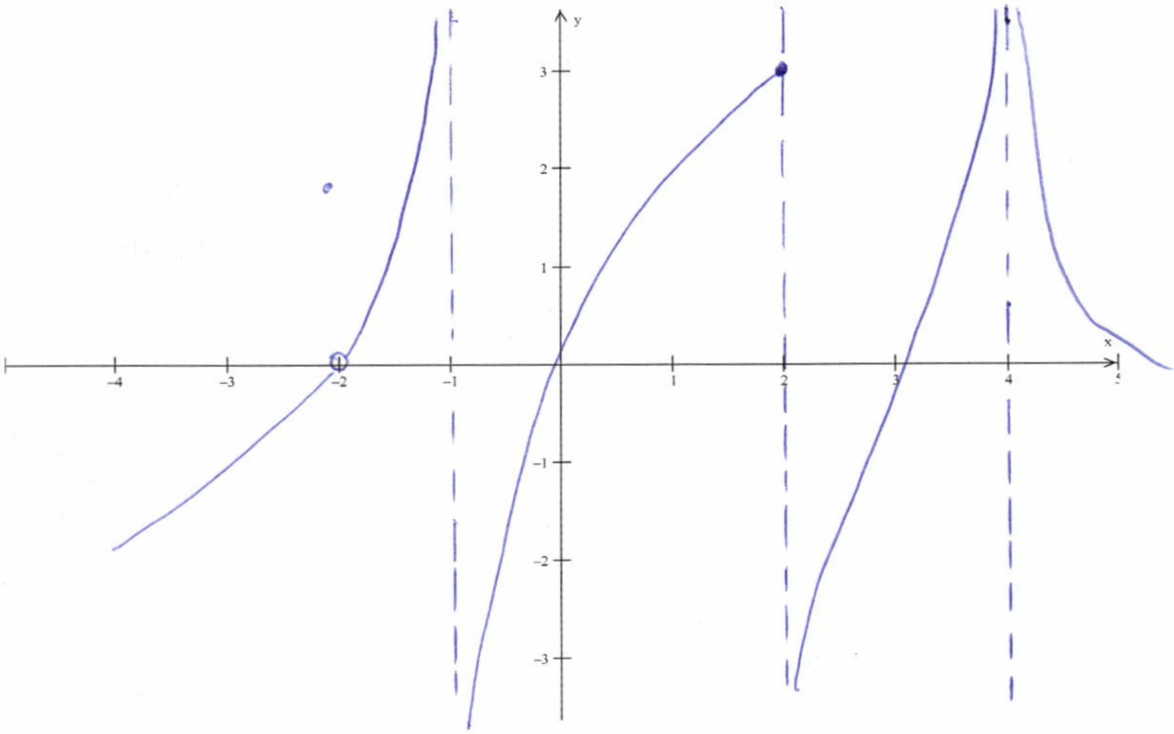
$\tan^{-1}\left[\lim\limits_{x\to 0^+}(x+\ln x)\right] = \tan^{-1}\left(0+\underbrace{\lim\limits_{x\to 0^-}\ln x}_{-\infty}\right)$

$= -\dfrac{\pi}{2}$

Problem 2: (4 points) Sketch the graph of a function f that satisfies all of the following conditions:

a. $\lim_{x\to 2^+} f(x) = -\infty$, $\lim_{x\to 2^-} f(x) = 3$, $\lim_{x\to -1^+} f(x) = -\infty$, $\lim_{x\to -1^-} f(x) = \infty$

b. $\lim_{x\to -\cancel{2}\,3} f(x) = \cancel{3}\,0$, $f(-\cancel{2}\,3) = \cancel{3}\,2$, $\lim_{x\to 4} f(x) = \infty$



Problem 3: (4 points) Use the $\epsilon - \delta$ definition of limit to show that $\lim_{x\to 1} (2-3x) = -1$.

Find values of δ that correspond to $\epsilon = 0.06$

Given $\epsilon > 0$, we need to find $\delta > 0$ such that if

$0 < |x-1| < \delta$ the $|(2-3x)-(-1)| < \epsilon$

$\iff |3-3x| < \epsilon$.

$\iff 3|x-1| < \epsilon$

$\iff |x-1| < \frac{\epsilon}{3}$

So we may take $\delta = \frac{\epsilon}{3}$.

Now if $\epsilon = 0.06$, we may take $\delta \le \frac{\epsilon}{3} = \frac{0.06}{3} = 0.02$

~~**Problem 4: (4 points)** Find all values of b which make the function continuous at 1.~~

Problem 4: **(4 points)** Find all values of b which make the function continuous at 1.

$$f(x)=\begin{cases} b-x^2 & \text{if } x>1 \\ \dfrac{b^2}{x}-3 & \text{if } x<1 \\ 4 & \text{if } x=1 \end{cases}$$

$\lim\limits_{x\to 1^+} f(x) = \lim\limits_{x\to 1^-} f(x)$.

$\Rightarrow \lim\limits_{x\to 1^+} (b-x^2) = \lim\limits_{x\to 1^-} \frac{b^2}{x}-3$

$\Rightarrow \quad b-1 = b^2-3$

or $b^2-b-2=0$.

$(b-2)(b+1)=0 \quad \Rightarrow b=2$ and $b=-1$.

If $b=2$, we see that $\lim\limits_{x\to 1^+}(2-x^2)=\lim\limits_{x\to 1^-}\left(\frac{4}{x}-3\right)=1 \neq f(1)$

But if $b=-1$ " " $\lim\limits_{x\to 1^+}(-1-x^2)=\lim\limits_{x\to 1^-}\left(\frac{1}{x}-3\right)$ so $-2=-4$ impossible

$\boxed{\therefore \text{ only } b=2}$

Problem 5: **(4 points)** Consider the function $f(x)=\dfrac{x^2-2x+1}{x^3-x}$.

a. Find all values of x where the function is discontinuous and state the type of each one.

$$\frac{(x-1)(x-1)}{x(x-1)(x+1)}$$

The function is discontinuous at

$x=0$ infinite disc.

$x=1$ removable disc.

$x=-1$ infinite disc.

b. Find all vertical asymptotes of the function.

Vertical asymptotes:

$\lim\limits_{x\to 0^+} f(x) = -\infty \quad \Rightarrow x=0$ is VA

$\lim\limits_{x\to 1} f(x) = \frac{0}{2}=0 \quad \Rightarrow x=1$ is not VA

$\lim\limits_{x\to -1^+} f(x) = \infty \quad \Rightarrow x=-1$ is VA

Problem 6: (4 points) If $\lim_{x\to 2} \frac{10+x-g(x)}{x-2} = 3$, find $\lim_{x\to 2} g(x)$. Justify your answer.

$$\lim_{x\to 2}[10+x-g(x)] = \lim_{x\to 2} \frac{10+x-g(x)}{x-2}(x-2)$$

$$= \lim_{x\to 2} \frac{10+x-g(x)}{x-2}(0)$$

$$= (3)(0) = 0.$$

But $0 = \lim_{x\to 2}(10+x-g(x)) = 10+2-\lim_{x\to 2} g(x)$

$\therefore \lim_{x\to 2} g(x) = 12.$

Problem 7: (4 points) Use the Intermediate Value Theorem to show that the two functions $\cancel{f}(x) = x^5+2$ and $\cancel{f}(x) = 5x^3-1$ intersect between $x=-1$ and $x=2$.

We need to show that there is a c between -2 and 2 such that $c^5+2 = 5c^3-1$.

Now we need $x^5+2 = 5x^3-1$

$x^5 - 5x^3 + 3 = 0.$

put $\cancel{f}(x) = x^5 - 5x^3 + 3.$

Here: (i) f is continuous on $[-1,2]$ (polynomial)

ii $f(-1) = -1+5+3 = 7 > 0$

$f(2) = 32 - 40 + 3 = -5 < 0.$

By the IVT, there is a no. c between -1 and 2 such that $f(c)=0$ which - point of intersection.